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EFFECT OF DIFFERENT CLUSTERING SCENARIOS ON DEMAND AND PRESSURE UNIFORMITY

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ABSTRACT

Clustering the network in district metering areas (DMA) has shown its usefulness in controlling water losses. However, the definition of the DMAs is highly conditioned by both topographic and hydraulic characteristics of the networks. In networks with enough hydraulic capacity, automatic definition of DMA is feasible. On the other hand, networks strongly constrained may have problem due to pressure and flow restrictions that should be solved using some engineering judgement.

In this paper, a comparison of both strategies is made. The methodology presented selects the approach depending on the hydraulic complexity of the network. So, in areas where the differences in elevation are not very high, an automatic graph partitioning algorithm will be used. On the contrary, where the hydraulic restrictions reduce the search space, engineering judgment is applied to find segmentation solutions that accomplish the different constraints. Finally, both methods are compared according different parameters, such as the demand or pressure similarity.

The main conclusion is that a appropriate combination of both approaches will lead to better results that those obtained by the use of each technique separately.

INTRODUCTION

Efficiency of water distribution networks is an increasingly important matter. Efficiency accounts for both energy and water resources, and water saving has become a main objective of the utilities. Since leakage can be treated as pressure dependent demands, pressure management has become a powerful tool for controlling non-revenue water losses (Araujo, Ramos, & Coelho 2006; Vairavamoorthy & Lumbers 1998). In this sense, clustering the network in district metering areas (DMA) has shown its usefulness in controlling water losses.

Traditionally, the partitioning of a network into DMAs was done using engineering judgement, that is, starting with a good knowledge of the network and acting accordingly to divide the network without conditioning its performance. Water networks may be understood as oriented graphs. For that reasons, some authors (Di Nardo & Di Natale 2011; Tzatchkov, Alcocer-Yamanaka, & Bourguett Ortíz 2008) use graph theory to define hydraulic sectors. If a network is interpreted as a series of nodes and arcs, algorithms for graph partitioning as METIS (Karypis & Kumar 1998) can be used. Di Nardo & Di Natale (2011) used this algorithm for network partitioning using energy criteria as objective. However, an automatic partitioning algorithm dose not account for hydraulic restrictions based on momentum and continuity equations. The existence of fixed demands to be delivered or pressure limits force to evaluate the partitioning obtained. In other words, the problem of DMA definition in water supply systems is a difficult problem.

For that reason, in the edition of the WDSA held in Cartagena (Colombia) in 2016, the Scientific Committee proposed the Battle of Water Networks District Metering Areas (BWNDMA Committee 2016). The aim of the contest consisted on produce a segmentation of the network so that the demand was equally distributed among the clusters (DMAs) and the pressure was as uniform as possible inside them.

Some of the participants chose a solution with a small number of clusters, prioritizing economic aspects (Gilbert et al. 2017). Some other groups preferred to define more clusters to ensure a more uniform distribution of demands and pressure (Martínez-Solano et al. 2018; Salomons, Skulovich, & Ostfeld 2017). All of them found that one of the problems in this contest was derived of its troubled orography. Furthermore, the restrictions related to water resources availability reduce the solution space. With these conditions, engineering judgment took advantage with respect to automatic clustering. There was some evidences that a relation between demand and pressure uniformities, and the number of DMA could be established.

This paper is aimed to show these evidences. To get that target, a comparison of both strategies is made. The methodology presented selects the methodology depending on the hydraulic characteristics of the network. That is, the selection of the clustering method is based on the hydraulic complexity of the network. So, in the areas where the differences in elevation is not very high, an automatic graph partitioning algorithm based on METIS (Karypis 2013) will be used. On the contrary, where the terrain is steep, the network layout is rigid or the restrictions limit the set of feasible solutions, engineering judgment needs to be applied to find segmentation solutions that accomplish the different constraints. Both methods are then compared using different parameters, such as the demand similarity among clusters, pressure uniformity inside them or capacity to accomplish with pressure limits.

This methodology is applied to the southern part of the E_Town network (BWNDMA Committee 2016). Engineering judgment will use hydraulic relations to choose the geometry of the clusters and the most suitable places to close pipes or install valves and flowmeter. On the other hand, a graph partitioning algorithm will be used in the places where the hydraulic restrictions are no so severe. As a result, the relation between the number of clusters and the different criteria is obtained. This relation might be used as a decision support tool for the utilities in cases where the problem is highly constraint.

This work compares the efficiency of the sectorization when both approaches are combined depending on the boundary conditions of the problem. So, in those cases where the restrictions allow a wide search space, an automatic algorithm for the sectorization based on the METIS algorithm show a good performance. However, in areas heavily restricted, an engineering-based approach allows reduce the search space around the feasible solutions. The paper applies this methodology to a part of the E-Town network (BWNDMA Committee 2016) where both scenarios are present.

Finally, different sectorization proposals were performed with different number of DMAs. These scenarios were compared attending to different quality indexes. As a result, a proposal for the partitioning that presents the best results is presented.

PROBLEM DESCRIPTION

Given a Water Distribution Network (WDN), the problem can be formulated as defining a new DMA configuration with a minimal number of DMAs, each with a similar demand. An additional outcome must be a pressure uniformity across the municipality and along the simulation period. Despite of the solution, the WDN must satisfy a set of restrictions such as maximum and minimum pressures, demand at nodes and the hydraulic relations of continuity and momentum equations.

A way to assess every solution for this sectorization of the WDN is the use of performance indexes. Usually, sectorization is used as a tool for the pressure management of the network. For this reason, one of the indexes should be a pressure uniformity index (PU). Besides, it is advisable that the network was divided into DMAs with similar sizes. A demand similarity index (DS) will account for this. Finally, if the network is highly restricted, it can be difficult to find feasible solutions. For this reason, the last index will be based on the number of nodes where the pressure restrictions are not met. Next, all these indices are presented.

Demand Similarity Index (DS)

The demand similarity is defined as the standard deviation of the demand supplied by the DMAs:

$$DS = \frac{1}{Q_{av}} \sqrt{\frac{1}{N_{DMA}} \sum_{i=1}^{N_{DMA}} (Q_i - Q_{av})^2}$$

In this equation, N_{DMA} is the number of DMAs, Q_i is the base demand supplied by the DMA i, and Q_{av} is the average demand of all DMAs, that is:

$$Q_{av} = \frac{1}{N_{DMA}} \sum_{i=1}^{N_{DMA}} Q_i$$

Pressure Uniformity of the network (PU_{Net})

The uniformity of pressures is one of the main objectives in the sectorization of a network. This uniformity might be understood from two different perspectives.

First, all nodes in the network must have a pressure as close as possible to a minimum admissible value. Low pressures imply better efficiency of the network in terms of leakage. Some authors Iglesias-Castelló, Iglesias-Rey, & Martínez-Solano (2018) state that low pressures lead to a minimum energy consumption in the WDN. In this sense, the closer the pressure to a fixed minimum value, the better is the sectorization. This could be measured as the difference between the pressure in the node (at every time step) and the minimum pressure:

$$PE = \sum_{t=1}^{T}\left[\frac{1}{N}\sum_{i=1}^{N}\left[\frac{p_i(t) - p_{min}}{p_{min}}\right]\right]$$

In this equation, PE is a pressure excess index, $p_i(t)$ is the pressure in the node i in the time t, T is the duration of the analysis (typically 24 hours), N is the number of nodes in the network, and p_{min} is the minimum pressure.

The second perspective involves the variability of pressures. All nodes within a DMA should have similar pressure. To evaluate the uniformity of pressures, a pressure uniformity index (PU) is defined:

$$PU = \sum_{t=1}^{T}\left[\frac{1}{p_{av}(t)}\sqrt{\frac{1}{N}\sum_{i=1}^{N}[p_i(t) - p_{av}(t)]^2}\right]$$

In the previous equation, $p_i(t)$ has already been defined, and $p_{av}(t)$ is the average pressure in the network at time t. $p_{av}(t)$ is calculated as:

$$p_{av}(t) = \frac{1}{N}\sum_{i=1}^{N} p_i(t)$$

Al-Hemairi & Shakir (2006) mixed both indexes to define a global pressure uniformity index for the WDN, PD_{Net}:

$$PU_{Net} = \sum_{t=1}^{T}\left[\frac{1}{N}\sum_{i=1}^{N}\left[\frac{p_i(t) - p_{min}}{p_{min}}\right] + \frac{1}{p_{av}(t)}\sqrt{\frac{1}{N}\sum_{i=1}^{N}[p_i(t) - p_{av}(t)]^2}\right]$$

So, the objective of the problem consists of finding the most suitable number of DMAs that minimizes both indexes (DS and PU_{Net}). The solution must satisfy the restrictions impose by the maximum and minimum pressures.

METHODOLOGY

In any WDN, there are parts of the network that work properly and parts that, due to topographical or hydraulic reasons, presents problems. These problems can be related to a series of reasons:

- Low pressures due to either altitude or hydraulic capacity of pipes
- High pressures, due to proximity to pumping stations, source tanks, or low elevation of nodes.
- Difficulties to meet demands.

Independently of the reason, solutions usually are based on engineering judgement. Khedr & Tolson (2016) propose the use of engineering judgement to look for feasible solutions. Taking these solutions as the starting point, some automatization algorithms can improve them. Iglesias-Rey et al. (2016) used the engineering judgement to fix the setting in pressure reducing valves (PRV) to minimize background leakage preserving the minimum pressure of the network where the network was strongly restricted. The methodology to proceed with the DMAs definition will depend on the capacity of the network to have feasible solutions. If the network is strongly restricted and is difficult to find feasible solutions, some engineering judgment can reduce the search space. On the contrary, if the network presents many feasible solutions, an automatic algorithm allows for selecting the most suitable according with some fixed criteria.

The methodology must start with a preliminary study. This study will allow to distinguishing areas conditioned by the feasibility of the hydraulic results from those with correct behavior in any situation. In this point, the approach for each type of area differs. Areas with difficulties to reach a solution satisfying all the restrictions will be processed using classical engineering techniques. Areas more suitable to present valid solutions will be applied the automatic approach.

Engineering judgement approach

In this approach, actions will be focused on the feasibility of the solution. The indexes are calculated but the decision about every action are based on the feasibility. The main rules used for this part are:

a) Classify the pipes in main pipes (bringing water from sources to tanks, reservoirs or pumps), transport pipes (pipes with high transport capacity and no demand nodes) and distribution pipes (pipes supplying water to users). Depending on this classification, pipes will be treated in one or another way.
b) Locate the nodes with problems due to low pressure. These nodes condition all the solutions and must be checked for every situation. All actions must be addressed to guarantee the pressure on them.
c) Locate the nodes with high pressure. In some occasions, this problem can be fixed by means of PRV. In some other cases, the solution will not be as easy, and a detailed study of the problem will be necessary.
d) Identify areas where the definition of the DMA is conditioned by the layout of the network. For example, this is the case of the branched networks.

Automatic heuristic optimization

In the areas where the hydraulic configuration allows multiple feasible solutions for the DMAs definition, an automated process is used. Even though the feasibility of the solutions is easy to reach, some engineering is also made in this approach. First, the classification of the pipes is used to define the DMA boundaries. Main pipes cannot be used as a boundary. The algorithm will be guided to avoid the use of transport pipes as boundaries. Finally, the aim is to leave this role to the distribution pipes.

The methodology presented for this type of networks will use the algorithm for graph partitioning developed by Karypis & Kumar (1998) known as METIS. For a given number of partitions, METIS will give a unique solution. This solution represents a set of nodes belonging to the same partition. However, METIS define the boundaries, as many of them as needed. METIS presents two features that can be very useful in this problem:

- Nodes can be weighted in such a way that the final solution has similar weight for all the partitions. This way, if the demand on the nodes is defined as their weight, the solution will have the best possible DS.

- Links (pipes) can also be weighted. METIS was designed for finite element grids and the original idea was to reduce the communication load among different processors. In the case of DMAs definition, link weights can be used to rank the pipes. So, distribution pipes will have a lower weight, increasing their possibilities to be chosen as boundaries. On the opposite side, main pipes will have a weight as high as possible. Hence, main pipes are not likely to be chosen as boundaries.

Usually, a DMA has one or two entries. To decide the status (open or closed) of the boundaries proposed by METIS, a genetic algorithm will be used. This algorithm will use the indexes presented above as objective function and the hydraulic restrictions will be defined through penalty functions. This way, unfeasible solutions are not completely discarded but will have a lower fitness and lower possibilities to transmit their genetic information to the next generation. As, for every number of DMAs, METIS will propose the partition with the best DS index, the objective function will be given by:

$$F = PU_{Net} + \lambda_0 + \lambda_1 \sum_{i=1}^{N} \sum_{t=1}^{T} [p_i(t) - p_{max}] + \lambda_2 \sum_{i=1}^{N} \sum_{t=1}^{T} [p_{min} - p_i(t)]$$

In this equation, $\lambda_1$takes a value of 0 if the pressure of node i at time step t is lower than the maximum and a penalty value otherwise. Same can be said about λ_2, but referred to minimum pressure. Finally, another penalty term (λ_0) is included to rank feasible solutions (with $\lambda_0 = 0$) better than unfeasible solutions.

CASE STUDY

To show the validity of the method, it was applied to the ETown WDN. This network had some restrictions the had to be accomplished:

- Water availability at sources were limited. For the purpose of this paper, this limitation was used only to decide the border between the area where engineering judgement was used and the area for automatic network partitioning.
- Pressure at nodes must be under 60 m.
- Minimum pressure at demand nodes must be over 15 m.
- The entrance to every DMA must have a PRV to adjust the pressure to levels as low as possible. As the cost of the PRVs depends on the size, it is advisable to choose small pipes as DMAs entry points.

The ETown WDN has two different operational areas with different topographic features. At the nord (South I in Figure 1), the network is almost flat, with elevations smaller of 10 meters. This makes easy to find DMAs with uniform pressures inside them. Besides, this part is supplied from a reservoir with enough water capacity. For this reason, this area was extended as far as possible. On the contrary, to the south (South II in Figure 1) there is a part of the network in the skirts of a mountain with elevations ranging from 0 to 63 meters. In some cases, this difference can be observed in the same line making feasibility according to pressure limits impossible. The supply of South II comes from a reservoir in the far soth of the network, with limited water capacity. So, Sotuh II will be defined as small as possible.

The left part of Figure 1 shows the Digital Elevation Model (DEM) of the network and the right part the proposal of division of the WDN in operational areas attending to the difficulties of defining DMAs.

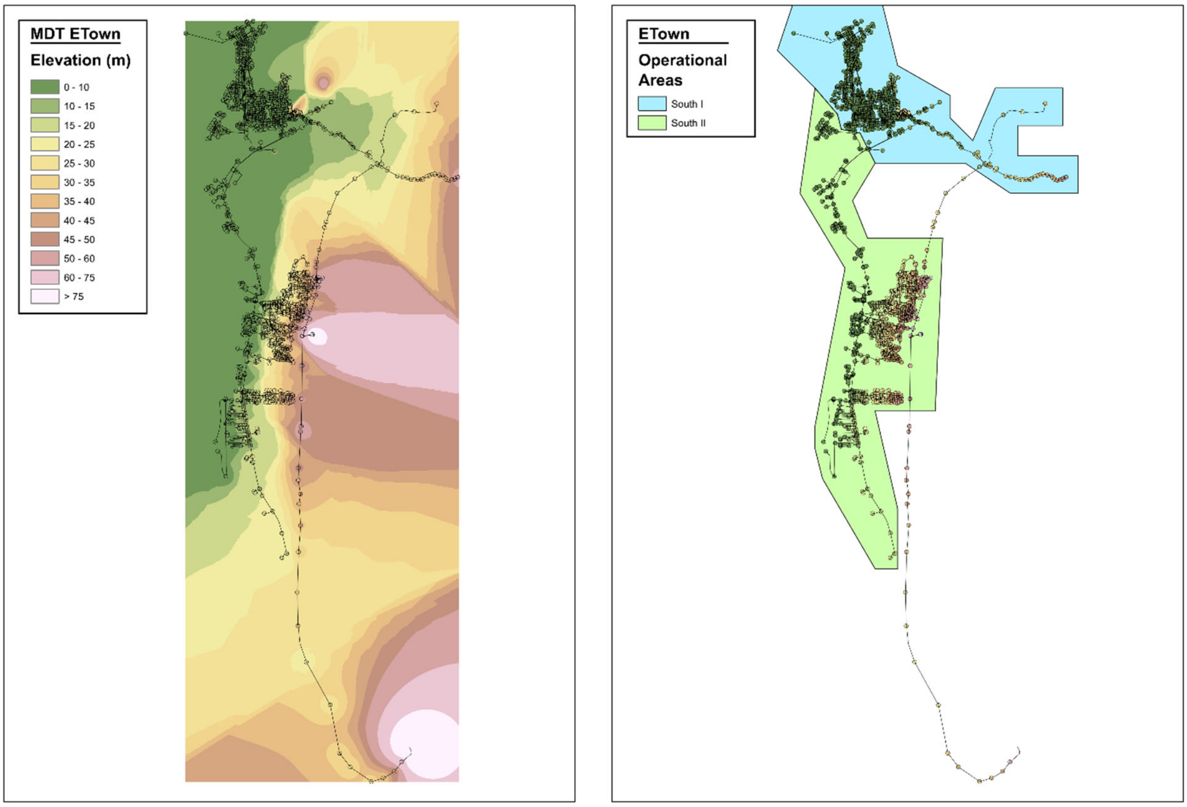

Figure 1. Digital Elevation Model and Operational Areas of ETown WDN.

After a preliminary analysis of the problem, a classification of the pipes was done attending to two criteria: the size of the pipes and the effects of using them as boundaries between DMAs. Three types of pipes were defined:

a) Main pipes. These pipes transport water from reservoirs to tanks or operational areas. If they are closed, some nodes will become unconnected to the reservoirs. These pipes never can be closed, then.

b) Transport pipes. They transport water from tanks to the entrances of the DMAs. If they are closed, some nodes will get also unconnected. However, they might be used as boundaries between DMAs only if they are open (and they have a PRV installed on them).

c) Distribution pipes. They are the smallest pipes in the network, usually with diameter up to 160 mm. They serve water to the consumers. From the sectorization point of view, they can present problems with transport capacity but their use as a boundaries is unexpensive.

The Figure 2 presents the classification of all pipes in these types.

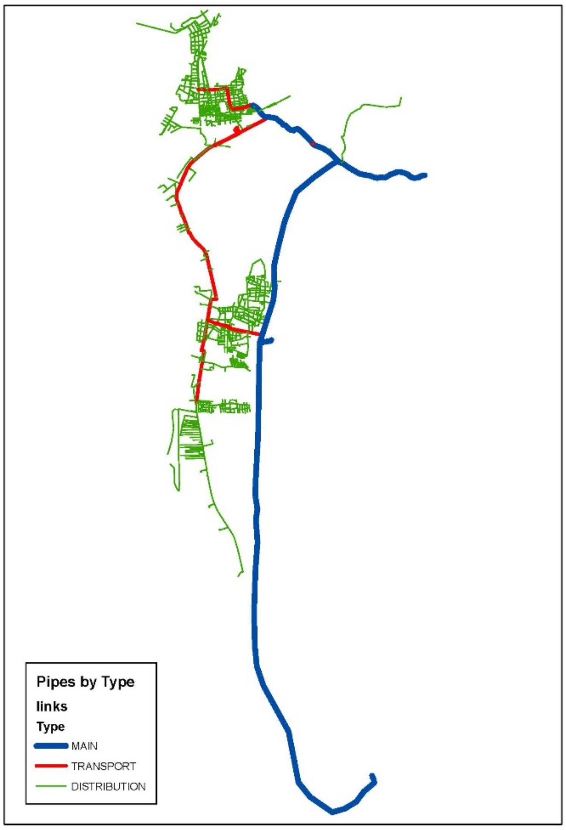

Figure 2. Classification of pipes according to their function in the network.

For the DMA definition of the network, two different strategies were assumed for each operational area:

Engineering judgement approach

In the South II area, physical knowledge of hydraulic phenomena was needed to solve problems related with the pressures. As an example, there is some parts in South II that have nodes with elevations of 12 and 47 meters in the same line (see Figure 3). In this cases, definition of DMA was done manually using the experience of authors.

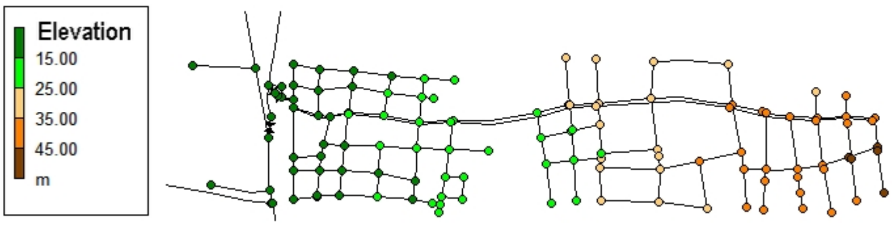

Figure 3. Example of topographic problems in South II área.

In these cases, the size of the DMA was not a criterion to be used since the main target was to accomplish with pressure limits.

Automatic heuristic optimization

Once the South II has been properly divided into DMAs, an average demand for the DMAs was got. This number is a starting point for the next step. For the South I part of the network, an automatic partitioning scheme was used. In this part of the network, there is no severe problems with pressures. For this reason, the partitioning algorithm used by METIS (Karypis 2013) was applied. This algorithm has some relevant features:

- It always reach the same solution for the same problem.
- It allows to weighting nodes. The algorithm distribute these weights among the nodes in such a way that the solution proposed has the best possible distribution of weights. Using nodal demands as weights ensures that the solution will present the best possible demand uniformity.
- It allows to weighting links. Link weights represented originally communication needs among processors and the algorithm looks for a solution with minimum communication requirements. In this case, weights has been defined depending on the type of pipe and its diameter so that transport and big pipes were less likely to be chosen as boundaries and small pipes were used instead. Each pipe was assigned a weight as shown in Table 1.

Table 1. Link weights used by the METIS algorithm.

D (mm)	Weight	D (mm)	Weight
50	22	300	18
60	17	350	25
70	14	400	32
80	13	450	41
90	11	500	50
100	1	600	72
150	2	700	98
200	4	1500	450
250	6	MAIN PIPES	500

The results produced by METIS consists of a list of nodes and their partition, that is, the DMA they belong to. A link connecting two nodes belonging to different partitions is a boundary between DMA. There can be as many boundaries as needed. Usually, a DMA must have one or two pipes allowing flow to get into it. Then, these supplying pipes must be selected among those proposed by METIS as boundaries. At this point, a genetic algorithm was used to look for the best combination that optimize the pressure uniformity and maintain pressure within the limits established by the restrictions previously presented.

This procedure was performed for different number of DMA assessing both the demand similarity and the pressure uniformity. Next, the results are discussed.

DISCUSSION OF RESULTS

In the first step, using engineering judgement allowed to reducing the number of nodes with excessive pressure. This number passed from 351 nodes to 16. Due to topographic restrictions, it was not possible to solve the pressure problems in those nodes. Figure 3 shows an example of the difficulties for solving this problems. Figure 4 shows the effect of the number of DMAs in the numbe of nodes with excessive pressure. It must be highligthed than the automatic partition approach does not affect to this parameter, maintaining this number in 16.

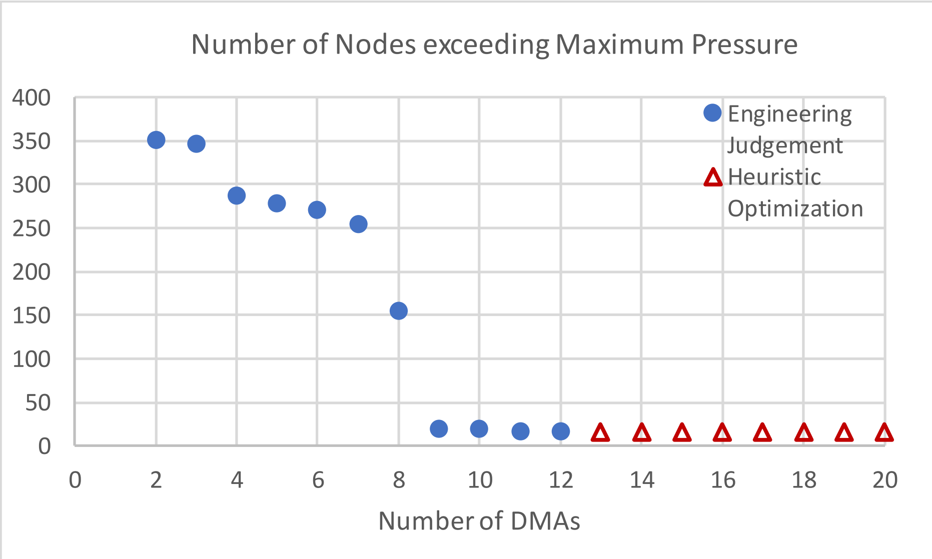

Figure 4. Unfeasibility. Number of nodes exceeding the máximum pressure.

Regarding the demand similarity, the engineering judgement approach does not help to minimize this parameter. The target of this approach was to solve problems with pressures and DS was hardly used in the process of DMA definition. That is the reason why the DS increases as the number of DMA does. However, once the heuristic optimization was used, the demand similarity was controlled. In fact, there is an optimal number of DMAs that minimizes the DS. The reason is that, due to the uneven distribution of demands, there is a point from which increasing the number of DMAs does not help reducing the DS parameter. Figure 5 shows the evolution of DS with the number of DMAs. From this figure, the conclusion is that 17 DMAs lead to the minimum value of DS.

Finally, the engineering judgement approach allowed to minimizing the pressure uniformty parameter. This approach was focused on pressure management and it is obvious that the result confirms this fact. As the number of DMAs was reducing, the problems due mainly to excessive pressure were decreasing and PU_{Net} did accordingly. Once the pressures were controlled in South II area, the engineering judgement approach limited its validity. When the automatic heuristic optimization was used, the pressure uniformity hardly changed due to the fact that this method included the setting of the PRV set at every entrance of a DMA. Fluctuation of the PU_{Net} with the number of DMA area almost negligible and might be explained by the heuristic nature of the algorithm. Figure 6 shows the evolution of PU_{Net} with the number of DMAs. In this figure all the previous comments can be observed.

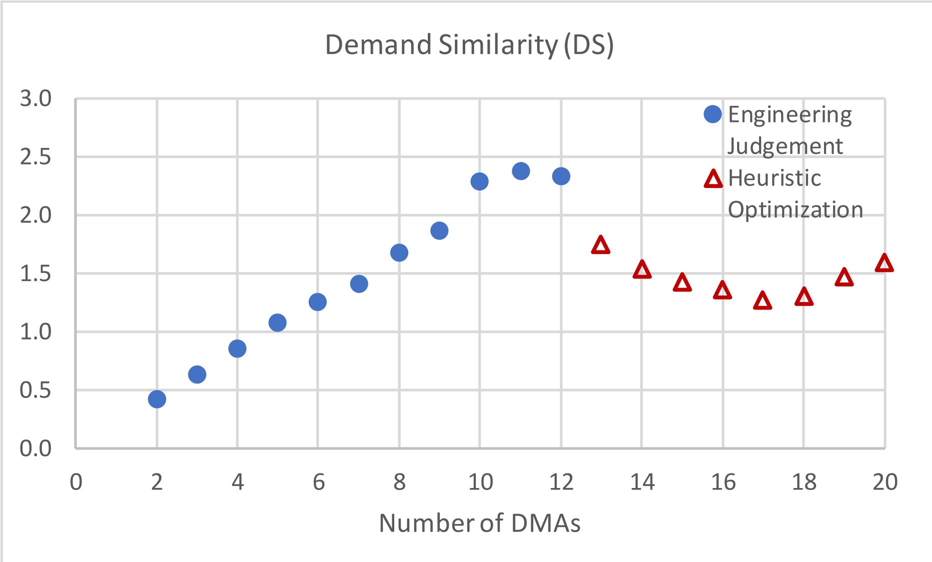

Figure 5. Evolution of demand similarity (DS) using both engineering judgement criteria and automatic network partition by means of heuristic optimization

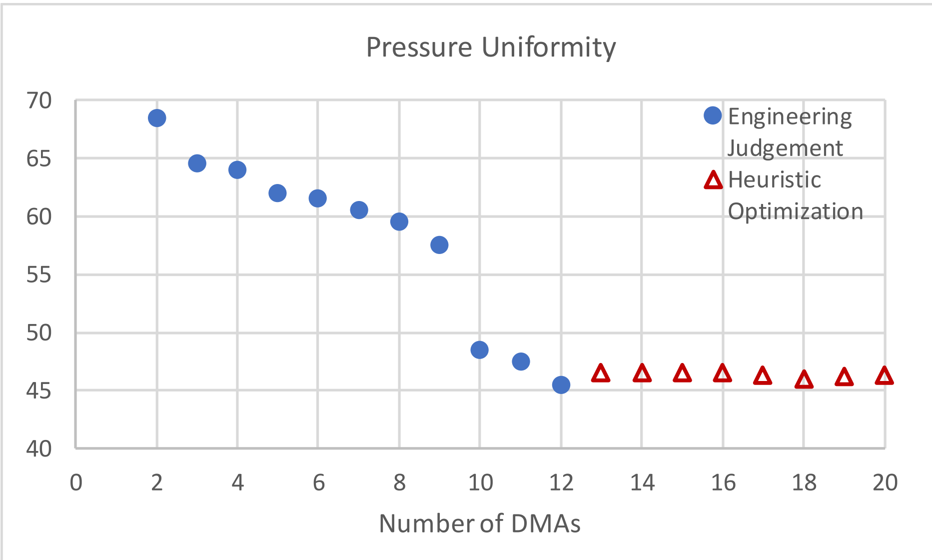

Figure 6. Evolution of pressure uniformity (PU_{Net}) using both engineering judgement criteria and automatic network partition by means of heuristic optimization

CONCLUSIONS

Network partitioning is a complex task. This task can be even harder when the restrictions reduce the search space of feasible solutions. For this reason, a hybrid method that combines automatic partitioning algorithms with engineering judgement have been tested in a strongly restricted network. The results prove the validity of the method either in feasibility of solutions and quality criteria selected for the DMAs.

After applying the method to the case study, some conclusions were arisen:

- Complete feasibility of the solution is impossible due to some restrictions. Flow capacity is very limited, and maximum and minimum pressures are difficult to accomplish in some areas due to topographical reasons.
- All solutions tested accomplished with the minimum pressure criteria. Besides, the use of engineering judgement in the areas with steep slopes (South II) allows to reducing the number of nodes with pressure higher than the maximum. However, it was not possible to find solutions with less than 16 nodes with excessive pressures.
- First, engineering judgement was used to define DMA so that pressure limitations was satisfied. This was necessary to be applied to the South part of the network (the area marked as South II in Figure 1). The number of DMAs was not a constraint, but a result of the process. As a result, the solution had as many DMAs as needed to reduce the number of nodes that had excessive pressure.
- In a second step, an automatic partitioning method was applied to the north part of the network since in this part of the network was flat and there was not problems with pressures. In this case, engineering judgement was used to assign weights to the pipes so that main pipes were hardly selected as entrances to DMA and transport pipes were selected according to the diameter, using economic criteria. Different number of DMAs was tried and assessed. As a result, the demand similarity parameter reached a minimum at 17 DMAs.
- With the automatic network partitioning, two other behavoir were observed. On one hand, the number of nodes with excessive pressure did not change. On the other hand, pressure uniformity remained almost unchanged. The latter was due mainly by the presence of PRV at every entrance of a DMA.

As a conclusion, it might be said that a clever combination of physical knowledge of the problem and powerful heuristic algorithms can bring better solutions than the use of any of them separately. Results of standard engineering solutions were improved by the use of automatic partitioning algorithms with a proper assessment of the solutions.

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REFERENCES

Al-Hemairi, H. A., & Shakir, R. H. (2006). Minimizing Leakage Rates in Water Distribution Networks through Optimal Valves Settings. In *World Environmental and Water Resource Congress 2006* (pp. 1–13). Reston, VA: American Society of Civil Engineers. http://doi.org/10.1061/40856(200)26

Araujo, L. S., Ramos, H., & Coelho, S. T. (2006). Pressure Control for Leakage Minimisation in

Water Distribution Systems Management. *Water Resources Management*, *20*(1), 133–149. http://doi.org/10.1007/s11269-006-4635-3

Committee, B. (2016). Battle of water networks DMAs (BWNDMA) — Problem description and rules. Retrieved June 16, 2016, from https://wdsa2016.uniandes.edu.co/index.php/battle-of-water-networks

Di Nardo, A., & Di Natale, M. (2011). A heuristic design support methodology based on graph theory for district metering of water supply networks. *Engineering Optimization*, *43*(2), 193–211. http://doi.org/10.1080/03052151003789858

Gilbert, D., Abraham, E., Montalvo, I., & Piller, O. (2017). Iterative Multistage Method for a Large Water Network Sectorization into DMAs under Multiple Design Objectives. *Journal of Water Resources Planning and Management*, *143*(11), 04017067. http://doi.org/10.1061/(ASCE)WR.1943-5452.0000835

Iglesias-Castelló, M., Iglesias-Rey, P. L., & Martínez-Solano, F. J. (2018). Potentially recoverable energy assessment in water distribution networks. In *WDSA/CCWI Joint Conference Proceedings*. Kingston, Canada.

Iglesias-Rey, P. L., Martínez-Solano, F. J., Mora Meliá, D., & Martínez-Solano, P. D. (2016). Combining Engineering Judgment and an Optimization Model to Increase Hydraulic and Energy Efficiency in Water Distribution Networks. *Journal of Water Resources Planning and Management*, *142*(5), C4015012. http://doi.org/10.1061/(ASCE)WR.1943-5452.0000605

Karypis, G. (2013). *METIS* A Software Package for Partitioning Unstructured Graphs , Partitioning Meshes , and Computing Fill-Reducing Orderings of Sparse Matrices. Version 5.1.0. Manual*. Minneapolis (MN). Retrieved from http://glaros.dtc.umn.edu/gkhome/fetch/sw/metis/manual.pdf

Karypis, G., & Kumar, V. (1998). Multilevel k-way Partitioning Scheme for Irregular Graphs. *Journal of Parallel and Distributed Computing*, *48*(1), 96–129. http://doi.org/10.1006/jpdc.1997.1404

Khedr, A., & Tolson, B. (2016). Comparing Optimization Techniques with an Engineering Judgment Approach to WDN Design. *Journal of Water Resources Planning and Management*, *142*(5), C4015014. http://doi.org/10.1061/(ASCE)WR.1943-5452.0000611

Martínez-Solano, F. J., Iglesias-Rey, P. L., Mora Meliá, D., & Ribelles-Aguilar, J. V. (2018). Combining Skeletonization, Setpoint Curves, and Heuristic Algorithms to Define District Metering Areas in the Battle of Water Networks District Metering Areas. *Journal of Water Resources Planning and Management*, *144*(6), 04018023. http://doi.org/10.1061/(ASCE)WR.1943-5452.0000938

Salomons, E., Skulovich, O., & Ostfeld, A. (2017). Battle of Water Networks DMAs: Multistage Design Approach. *Journal of Water Resources Planning and Management*, *143*(10), 04017059. http://doi.org/10.1061/(ASCE)WR.1943-5452.0000830

Tzatchkov, V. G., Alcocer-Yamanaka, V. H., & Bourguett Ortíz, V. (2008). Graph Theory Based Algorithms for Water Distribution Network Sectorization Projects. In *Water Distribution Systems Analysis Symposium 2006* (pp. 1–15). Reston, VA: American Society of Civil Engineers. http://doi.org/10.1061/40941(247)172

Vairavamoorthy, K., & Lumbers, J. (1998). Leakage Reduction in Water Distribution Systems: Optimal Valve Control. *Journal of Hydraulic Engineering*. http://doi.org/http://dx.doi.org/10.1061/(ASCE)0733-9429(1998)124:11(1146)