Assessment of Regime Equations for Predicting General Scour

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ABSTRACT

There are various equations for predicting general scour that are presented in technical papers, design guidelines, and design manuals. Three such equations are the regime equations which were originally published by Gerald Lacey in 1930, Thomas Blench in 1969, and C.R. Neill in 1973. These regime equations are included in a United States Bureau of Reclamation (USBR) technical guideline titled, "Computing Degradation and Local Scour", 1984; National Resources Conservation Service (NRCS), National Engineering Handbook (NEH) Part 654 - Technical Supplement 14B, Scour Calculations, 2007; and in various local agency design manuals. However, the regime equations as presented in these publications are different in form than the equations in the original reference documents and as a result provide different calculated scour depths. The paper examines the differences between the equations as reported in the design guidelines/manuals and the equations as originally documented, summarizes limitations of the equations, and provides examples to compare the variation in scour depth when applying different forms of the equations.

INTRODUCTION

The United States Bureau of Reclamation (USBR), Engineering and Research Center, published a technical guideline in January 1984 titled, "*Computing Degradation and Local Scour*" (Pemberton and Lara 1984). The purpose, as stated in the introduction to the guide, was to present several methods which could be applied in computing degradation of a stream channel occurring because of changes in flow regimen or reduced sediment load below a dam or diversion dam, and to provide procedures to use in estimating maximum scour depth of channels for design of a structures such as a bridge or siphon crossing. The guideline provides a compilation of procedures for evaluating: 1) degradation limited by armoring; 2) degradation limited by a stable slope; and 3) channel scour during peak flood flows.

The USBR guideline for channel scour includes three regime equations originally presented by Gerald Lacey (Lacey 1930), Thomas Blench (Blench 1969), and C.R. Neill (Neill 1973) and therefore referenced as the Lacey Equation, Blench Equation, and Neill Equation. These three equations, as presented by the USBR, have been adopted by various federal and local agencies as a means for computing general scour in channels.

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The National Resources Conservation Service (NRCS) included the Lacey and Blench Equations in Part 654 of the National Engineering Handbook (NEH) – Technical Supplement 14B, Scour Calculations (NRCS 2007). NRCS noted that the USBR suggested that regime equations provided by Blench and Lacey could be used to predict general scour in natural channels. NRCS expressed these two regime relationships, as presented by the USBR, as one relationship with a constant and exponents that were dependent upon the equation being used, Lacey or Blench, and the channel condition.

A comparison of the equations offered by the USBR with the original regime equations presented in the reference documents by Lacey, Blench, and Neill shows the equations are different for each of the three equations. Therefore, the scour depth calculated by the USBR form of the equations is different than the scour depth computed by applying the equations presented in the reference documents prepared by Lacey, Blench, and Neill.

The original relationships presented by Lacey, Blench, and Neill were developed to predict the scoured depth which is defined as the depth measured between the water surface elevation and the scour elevation. The USBR as well as the NRCS equations predict the scour depth or the depth measured between the minimum channel elevation and the scour elevation This paper compares the USBR and NRCS regime equations with the regime equations as originally developed by Lacey, Blench, and Neill.

LACEY EQUAITON

<u>USBR (Pemberton and Lara 1984)</u>

The USBR form of the Lacey Equation is given as:

$y_m = 0.47 (Q / f)^{1/3}$

where:

y_m = mean water depth at design discharge, (ft)
Q = design discharge, (ft^3/s)
f = Lacey's silt factor = $1.76 (D_m)^{1/2}$
D_m = mean grain size of bed material, (mm)

The scour depth below the streambed is calculated from the following equation.

$y_s = Z\, y_m$

where:

y_s = scour depth below low point of stream bed, (ft)
y_m = mean water depth at design discharge, (ft)
Z = multiplying factor from USBR Table 7

The multiplying factors given by USBR Table 7 for the Lacey Equation are as follows:

Stream Condition	*Value of Z*
Straight reach	0.25
Moderate bend	0.5
Severe bend	0.75
Right angle bends	1.0
Vertical rock bank or wall	1.25

NRCS (NRCS 2007)

The NRCS form of the Lacey Equation is given as:

$$y_s = K\, Q_d^{a}\, W_f^{b}\, D_{50}^{c}$$

where:

y_s = maximum scour depth, (ft)
K = coefficient (table TS14B-8)
a, b, c = exponents (table TS14B-8)
Q_d = design discharge, (ft^3/s)
W_f = flow width at design discharge, (ft)
D_{50} = median size of bed material, (mm)

The constant and exponents given by NEH Table TS14B-8 for the Lacey Equation are as follows:

Stream Condition	K	a	b	c
Straight reach	0.097	1/3	0	-1/6
Moderate bend	0.195	1/3	0	-1/6
Severe bend	0.292	1/3	0	-1/6
Right angle bends	0.389	1/3	0	-1/6
Vertical rock wall	0.487	1/3	0	-1/6

Lacey (Lacey 1930)

The regime equation as presented by Lacey is:

$$R = 0.47\,(Q / f)^{1/3}$$

where:

R = hydraulic depth at design discharge, (ft)
Q = design discharge, (ft^3/s)
ƒ = Lacey's silt factor = $1.76\,(D_m)^{1/2}$
D_m = mean grain size of bed material, (mm)

However, it is noted the above equation only applies when the channel width is equal to the regime width. The regime width as defined by Lacey is equal to 2.67 $Q^{1/2}$. For widths other than the regime width, the equation is:

$$R = 0.91\,(q^2 / f)^{1/3}$$

where:

R = hydraulic depth at design discharge, (ft)
q = unit design discharge, (ft^3/s/ft)
f = Lacey's silt factor = 1.76 $(D_m)^{1/2}$
D_m = mean grain size of bed material, (mm)

The maximum scoured depth (water-surface elevation – scour elevation) is determined by the following equation.

$$D_{max} = c\,R$$

where:

D_{max} = maximum scoured depth, (ft)
R = hydraulic depth at design discharge, (ft)
c = multiplying factor from Lacey

The multiplying factors given by Lacey are as follows:

Stream Condition	*Value of c*
Straight reach	1.27
Moderate bend	1.5
Severe bend	1.75
Right angle bends	2.0

Once the maximum scoured depth has been determined, the scour depth below the streambed can be calculated as:

$$y_s = D_{max} - y_{max}$$

where:

y_s = scour depth below low point of stream bed, (ft)
D_{max} = maximum scoured depth for the design discharge, (ft)
y_{max} = maximum water depth for the design discharge, (ft)

Discussion of Lacey Equation

The equation as presented by the USBR and the NRCS uses the design discharge (Q) rather than the unit design discharge (q). For a given discharge (Q), Manning's "n", and mean particle diameter (D_m), the equations predict the same scour depth whether the channel is 100 feet wide or 1,000 feet wide and whether the channel slope is 0.5% or 3.0%.

The regime equations as documented by Lacey, Blench, and Neill result in predicting the maximum scoured depth (water-surface elevation – scour elevation), not the scour depth. The scour depth can then be calculated as the maximum scoured depth minus the maximum water depth.

BLENCH EQUATION

USBR (Pemberton and Lara 1984)

The USBR form of the Lacey Equation is given as:

$$y_{f0} = q_f^{2/3} / F_{b0}^{1/3}$$

where:

y_{f0} = depth for zero bed sediment transport, (ft)
q_f = design flood discharge per unit width, ($ft^3/s/ft$)
F_{b0} = Blench's "zero bed factor", (ft/s^2)
from USBR Figure 9 - Chart for estimating F_{b0}
D_m = median bed-material size by weight, (mm)

The scour depth below the streambed is calculated as:

$$y_s = Z\, y_{f0}$$

where:

y_s = scour depth below low point of stream bed, (ft)
y_{f0} = water depth for zero bed sediment transport, (ft)
Z = multiplying factor from USBR Table 7

The multiplying factors given by USBR Table 7 for the Blench Equation are as follows:

Stream Condition	*Value of Z*
Straight reach	0.6 [1]
Moderate bend	0.6 [1]
Severe bend	0.6 [1]
Right angle bends	1.25

[1] *Z value selected by USBR*

NRCS (NRCS 2007)

The NRCS form of the Blench Equation is given as:

$$y_s = K\, Q_d^{a}\, W_f^{b}\, D_{50}^{c}$$

where:

y_s = maximum scour depth, (ft)
K = coefficient (table TS14B-8)
a, b, c = exponents (table TS14B-8)
Q_d = design discharge, (ft^3/s)
W_f = flow width at design discharge, (ft)
D_{50} = median size of bed material, (mm)

The constant and exponents given by NEH Table TS14B-8 for the Blench Equation are as follows:

Stream Condition	K	a	b	c
Straight reach	0.530	2/3	-2/3	-0.1092
Moderate bend	0.530	2/3	-2/3	-0.1092
Severe bend	0.530	2/3	-2/3	-0.1092
Right angle bends	1.105	2/3	-2/3	-0.1092

Blench (Blench 1969)

The regime equation as presented by Blench is:

$$y_{f0} = q_f^{2/3} / F_{b0}^{1/3}$$

where:

y_{f0} = scoured depth, (ft)
q_f = design discharge per unit width, ($ft^3/s/ft$)
F_{b0} = Blench's "zero bed factor", (ft/s^2)
  $F_{b0} = 1.9\, D_m^{0.5}$ $D_m \le 2.0$ mm
  $F_{b0} = 1.75\, D_m^{0.25}$ $D_m > 2.0$ mm
D_m = median bed-material size by weight, (mm)

To calculate the maximum scoured depth for design, D_{max}, the scoured depth is multiplied by a factor that takes into account the angle of attack on a structure.

$$D_{max} = c\, y_{f0}$$

where:

D_{max} = maximum scoured depth, (ft)
y_{f0} = scoured depth, (ft)
c = multiplying factor from Blench

The multiplying factors given by Blench Equation are as follows:

Stream Condition	*Value of c*
Severest attack on natural meander bend	1.7
Abrupt impingement of flow on long bank	2.0 - 2.25

The scour depth is computed from the following equation:

$y_s = D_{max} - y_{max}$

where:

y_s = scour depth, (ft)
D_{max} = maximum scoured depth, (ft)
y_{max} = maximum water depth, (ft)

Discussion of Blench Equation

Blench does not address different bend conditions in his discussion on scour and therefore does not provide a 'c' factor for straight reach, moderate bend, or severe bend conditions. It is implied that a 'c' factor of 1.0 be applied other than for the severest attack on natural meander bends and abrupt impingement of flow on a long bank. The USBR recommended a 'Z' factor of 0.6 for straight reach, moderate bend, and severe bend conditions. The USBR table listing the 'Z' factors includes a footnote stating, "Z value selected by USBR for use on bends in river".

The USBR provides a chart for determining Blench's 'zero bed factor', F_{b0}, whereas Blench provides the following relationships:

$F_{b0} = 1.9\ D_m^{0.5}$ for $D_m \leq 2.0$ mm; and
$F_{b0} = 7.3\ D_m^{0.25}$ for $D_m > 0.0066$ ft. (For millimeters, $F_{b0} = 1.75\ D_m^{0.25}$ for $D_m > 2.0$ mm.)

Neill Equation

USBR (Pemberton and 1984)

The USBR form of the Neill Equation is given as:

$y_f = y_i\ (q_f / q_i)^m$

where:

y_f = scoured depth below design floodwater level, (ft)
y_i = average depth at bankfull discharge, (ft)
q_f = design flood discharge per unit width, (ft^3/s/ft)
q_i = bankfull discharge per unit width, (ft^3/s/ft)
m = exponent – from 0.67 (sand) to 0.85 (coarse gravel)

The scour depth below the streambed is calculated as:

$y_s = Z\, y_f$

where:

y_s = scour depth below low point of stream bed, (ft)
y_f = scoured depth below design floodwater depth, (ft)
Z = multiplying factor from USBR Table 7

Stream Condition	*Value of Z*
Straight reach	0.5
Moderate bend	0.6
Severe bend	0.7

Neill (Neill 1973)

The regime equation as presented by Neill is

$y_f = y_i\, (q_f / q_i)^m$

where:

y_f = average scoured depth below design floodwater level, (ft)
y_i = average depth at bankfull discharge, (ft)
q_f = design flood discharge per unit width, ($ft^3/s/ft$)
q_i = bankfull discharge per unit width, ($ft^3/s/ft$)
m = exponent – from 0.67 (sand) to 0.85 (coarse gravel)

The depth of scour below the streambed is calculated as:

$y_s = D_{max} - y_{max}$

where:

y_s = scour depth below low point of stream bed, (ft)
D_{max} = maximum scoured depth below design flood level, (ft)
$1.4\, y_f < D_{max} < 1.7\, y_f$
y_{max} = maximum depth of flow, (ft)

Discussion of Neill Equation

Neill's Equation applies where a bridge constricts or realigns natural flows such that an artificial waterway opening is in effect created, bounded on each side by road approach embankments or guide banks.

The equation estimates the scour depth due to design flow passing through the controlled waterway opening. Neill's procedure is an iterative procedure whereas the USBR provides an equation for a direct solution.

EXAMPLE – LACEY AND BLENCH EQUATIONS

The USBR and the NRCS forms of the Lacey Equation and Blench Equation provide different results than the original form of the equations reported by Lacey and Blench. To demonstrate the difference in calculated scour depths and address issues associated with the different equation forms, a trapezoidal channel section with 2H:1V side slopes is selected. The scour depth is calculated for four different conditions using channel widths of 200 feet and 400 feet, and channel slopes of one-half percent (0.5%) and three percent (3.0%).

The channels each include a low-flow channel two feet in depth with a ten-foot bottom width and 2H:1V side slopes. The hydraulic and scour calculations are based on a Manning's "n" value of 0.035, a design discharge of 10,000 cubic feet per second (cfs), and a mean grain size of bed diameter of 2 mm. Figure 1 shows the typical section for each channel condition along with hydraulic parameters. The width, W_T, given is the width at one-half the flow depth. The hydraulic parameters for the four channel conditions are given in Table 1.

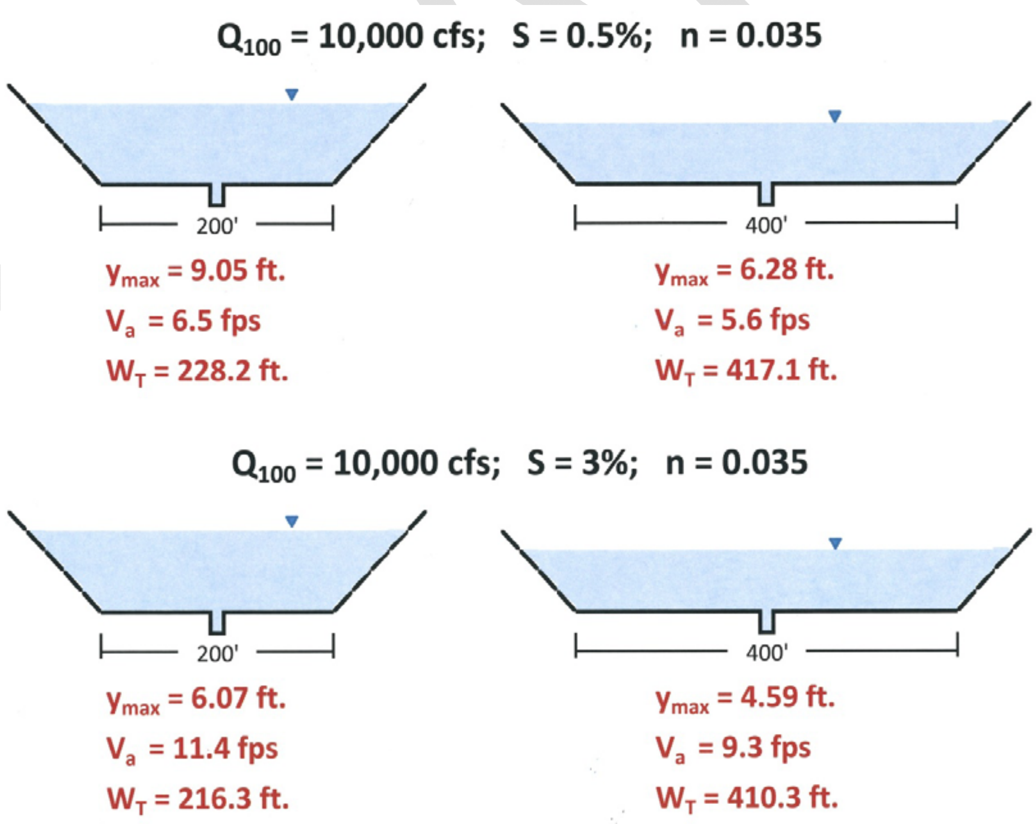

Figure 1. Channel Sections

Table 1. Hydraulic Conditions

Design Discharge (cfs)	Channel Slope (%)	Bottom Width (ft)	Maximum Flow Depth (ft)	Average Velocity (fps)
10,000	0.5	200	9.05	6.5
10,000	3.0	200	6.07	11.4
10,000	0.5	400	6.28	5.6
10,000	3.0	400	4.59	9.3

The scour depths calculated by the various forms of the Lacey Equation are given in Table 2 and the scour depths calculated by the various forms of the Blench Equation are given in Table 3.

<u>Results - Lacey Equation</u>

Table 2. Scour Depth – Lacey Equation

Channel Slope (%)	Channel Width (ft)	Average Velocity (fps)	Scour Depth – Lacey Equation (ft)		
			USBR	NRCS	Lacey
0.5	200	6.5	1.86	1.86	1.53
3.0	200	11.4	1.86	1.86	4.89
0.5	400	5.6	1.86	1.86	0.80
3.0	400	9.3	1.86	1.86	2.57

As can be seen in Table 2, the scour depth calculated by the USBR and NRCS forms of the Lacey Equation is the same for each of the four channel conditions. As noted above, the USBR and NRCS equations use the design discharge (Q) rather than the unit design discharge (q). With the same discharge (Q), Manning's "n", and mean particle diameter (D_m) used in each case, the scour depth is the same whether the channel is 200 feet wide or 400 feet wide and whether the channel slope is 0.5% or 3.0%. The equation as presented by Lacey utilizes the unit discharge resulting in a different scour depth for each of the four conditions. The scour depth calculated by the equation presented by Lacey is different for each of the four conditions with the scour depth increasing as the average velocity increases.

<u>Results - Blench Equation</u>

Table 3. Scour Depth – Blench Equation

Channel Slope (%)	Channel Width (ft)	Average Velocity (fps)	Scour Depth – Blench Equation (ft)		
			USBR	NRCS	Blench
0.5	200	6.5	5.71	6.11	0.29
3.0	200	11.4	5.61	6.34	3.45
0.5	400	5.6	3.66	4.09	0.00
3.0	400	9.3	3.64	4.13	1.52

Blench's Equation uses the unit design discharge rather than design discharge so the calculated scour depths for the USBR and NRCS forms of the equation are not all the same as was the case with the Lacey Equation. However, it is noted that since the unit discharge is the same or nearly the same for a given channel width, the calculated scour depth is essentially the same for both a channel slope of 0.5% and slope of 3.0%. The difference shown in Table 3 is the result of the unit discharge based on the average channel width rather than the using either the bottom width or top width when calculating the unit discharge. The scour depths calculated from the equation as presented by Blench are different for each channel condition since the scour depth is determined by subtracting the maximum flow depth from the maximum scoured depth rather than multiplying the maximum scoured depth by a constant which is the procedure for the USBR and NRCS forms of the Blench Equation. As was the case with the Lacey Equation, the scour depth calculated by the equation presented by Blench is different for each of the four conditions with the scour depth increasing as the average velocity increases.

EXAMPLE – NEILL EQUATION

As stated above, Neill's Equation applies where a bridge constricts or realigns natural flows such that an artificial waterway opening is in effect created, bounded on each side by road approach embankments or guide banks. Therefore, the equation is used for computing contraction scour. The example provided below is taken from the 1973 publication by Neill.

The upstream channel is in an incised reach having a bankfull discharge of 30,000 cfs. The channel is a trapezoidal channel with 2.5H:1V side slopes, a bottom width of 750 feet, a top width of 800 feet, and a flow depth of 10 feet. The channel section at the bridge is a trapezoidal channel with 2.5H:1V side slopes, a bottom width of 850 feet, and an initial estimated scoured depth of 30 feet and top width of 1000 feet. The design discharge at the opening is 150,000 cfs. Figure 2 is a schematic of the upstream (incised) section and the downstream (waterway opening) section.

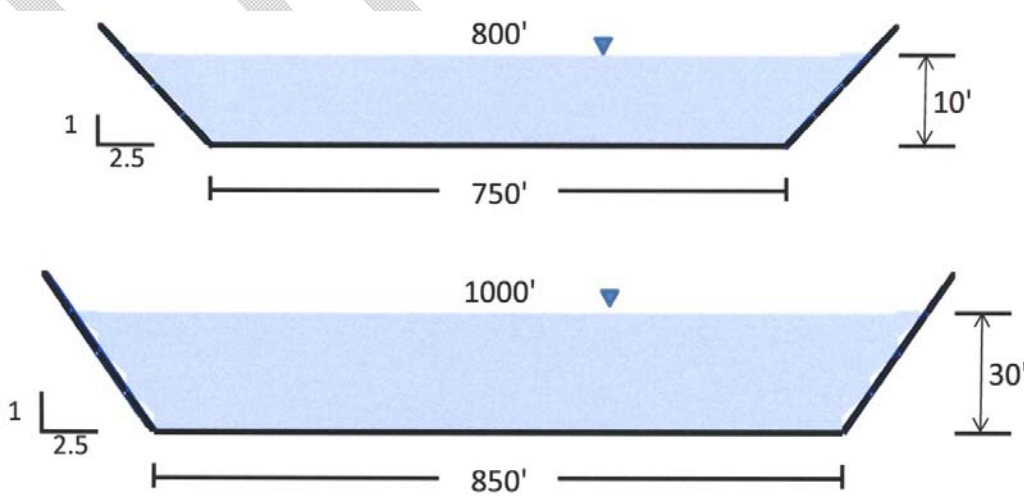

Figure 2. Upstream and Downstream Sections

The scour depth computed by the two forms of the Neill Equation is provided in Table 4. The depth computed by the USBR form of the equation is a direct solution whereas the depth computed from Neill's original form is an iterative procedure.

Table 4. Scour Depth – Neill Equation

Channel Section	Design Discharge (cfs)	Channel Width (ft)	Flow Depth (ft)	Scour Depth (ft)	
				USBR	Neill
Upstream	30,000	750	10	13.1	10.8
Downstream	150,000	850	30		

SUMMARY

The regime equations of Lacey and Blench have several limitations. Field data used to develop the equations was from Indian irrigation canals with sand beds and slightly cohesive-to-cohesive banks. The limitations include:

a. Steady discharge,
b. Steady bed-sediment discharge of too small an amount to appear explicitly in the equation,
c. Duned sand bed of natural particle size distribution and 0.1 mm < d_{50} ≤ 0.6 mm,
d. Suspended load insufficient to affect the equation,
e. Straightness in plan, so that the smoothed duned bed is level across the cross-section,
f. Uniform channel slope and section,
g. Flows of low Froude numbers in the ripple to dune range,
h. Flows remain in the channel, does not reflect the effects of floodplain flows in channel formation.

The regime equations presented by Lacey, Blench, and Neill calculate the scoured depth which is the maximum water depth including scour (i.e., depth measured between the design water surface elevation and the scour elevation). The maximum scoured depth is then determined by applying a multiplying factor to the depth calculated from the given regime equation. The procedures presented by the USBR and NRCS calculate a scour depth rather than a scoured depth. The USBR and NRCS methods calculate the scour depth by applying a multiplying factor that is one (1) unit less than the multiplying factor recommended by Lacey, Blench, and Neill. This procedure appears to assume the scoured depth is equal to the maximum water depth for the given design discharge which is unlikely. The resulting scour depth is different than the scour depth obtained by subtracting the maximum water depth from the calculated maximum scoured depth.

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