# Application of an analytical model to obtain daily flow duration curves for different hydrological regimes

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**Abstract**

This work assesses the performance of an analytical model framework to generate daily flow duration curves, FDCs, based on climatic characteristics of the catchments and on their streamflow recession coefficients. According to the analytical model framework, precipitation is considered to be a stochastic process, modelled as a marked Poisson process, and recession is considered to be deterministic, with parameters that can be computed based on different models. The analytical model framework was tested for three case studies with different hydrological regimes located in Switzerland: pluvial, snow-dominated and glacier. For that purpose, five time intervals were analyzed (the four meteorological seasons and the civil year) and two developments of the model were tested: one considering a linear recession model and the other adopting a nonlinear recession model. Those developments were combined with recession coefficients obtained from two different approaches: forward and inverse estimation. The performance of the analytical framework when considering forward parameter estimation is poor in comparison with the inverse estimation for both, linear and nonlinear models. For the pluvial catchment, the inverse estimation shows exceptional good results, especially for the nonlinear model, clearing suggesting that the model has the ability to describe FDCs. For the snow-dominated and glacier catchments the seasonal results are better than the annual ones suggesting that the model can describe streamflows in those conditions and that future efforts should focus on improving and combining seasonal curves instead of considering single annual ones

# 1 Introduction

A flow duration curve, FDC, is a common representation of the availability and variability of the daily discharges in a given river section. It is useful for many engineering applications, such as the design of small hydropower plants or water supply systems, studies about river ecology alterations and sediment transport and water quality and allocation (Vogel and Fennessey, 1994).

The characterization of the hydrological regime of a river, one of the goals of a FDC, is particularly important for run-of-river hydropower plants, that are very sensitive to natural fluctuations in the streamflows because they do not have a reservoir for water storage. Hydropower plants are usual in mountainous regions, that can be affected by snow and glacier processes, so modelling the regimes in catchments subject to those processes and potentially useful for hydropower production is increasingly important in a context of climate change.

FDCs can be obtained empirically, by assigning empirical probabilities to observed ranked daily discharges (Vogel and Fennessey, 1994), or by using models that generate the curves based on hydrological variables other than discharges (Castellarin et al., 2013). An important category of the models able to estimate FDCs are the process-based ones that combine climate controls and catchment characteristics. They can be based on long-term simulations of daily discharges or on the parametrization of the curves calculated from few key hydrologic controls, such as the model framework considered in this study.

Botter et al. (2007c) assumed that precipitation can be modeled as a stochastic process (i.e. a marked Poisson process) and be combined with a linear recession model to obtain FDCs that follow a gamma distribution. This approach only requires a few parameters: the mean depth of the daily precipitation, the frequency of the precipitation events that produce discharge, the area and the mean residence time (i.e., the inverse of the linear recession coefficient) of the catchment. This original framework has already been extended to embrace other hydrological conditions, such as nonlinear recession models (Botter et al., 2009), snow accumulation during winter (Schaefli et al., 2013), urbanized catchments (Mejía et al., 2014) and dry climates (Müller et al., 2014). The original model framework and its extensions have already been applied successfully to catchments in the US (Basso et al., 2015; Botter et al., 2013; Ceola et al., 2010), Italy (Botter et al., 2007b), Switzerland (Basso et al., 2015; Santos et al., 2018a; Schaefli et al., 2013), Nepal (Müller et al., 2014) and Portugal (Santos et al., 2018b). Previous applications in general avoid catchments with snow processes and, particularly in Switzerland, they did not consider a full year, but instead, specific seasons, namely summer in different hydrological regimes (Santos et al., 2018a) and winter in snow dominated catchments (Schaefli et al., 2013). Despite the large applicability of the model over different regions, Switzerland is interesting as a case study because it presents a variety of climates ranging from pluvial to glacier and has easily available and reliable data.

Process based frameworks, such as the one under study, have the advantages of providing an explicit link between the FDC shape, rainfall characteristics and catchment recession characteristics and of being applicable to periods characterized by different meteorological conditions. Besides, this framework is also simple and does not require much data or computational capacity.

The objective of the present work is to test the performance of an analytical model framework to obtain seasonal and annual FDCs in three Swiss catchments: Murg at Wangi, Grosstalbach and Rhone at Gletsch river gauge stations. The applications considered as time intervals to which the FDCs refer the four meteorological seasons and the civil year. The recession parameters were calculated based on the assumption of linear and nonlinear recession models. The calculation of the parameters followed two different approaches: a forward estimation, which consists in computing the parameters directly from the data, and the inverse estimation, which applies a calibration procedure thus resulting in optimized parameters.

The paper is organized as follows: Section 2 provides a description of the analytical model framework, together with the methods adopted, followed by the presentation of the case studies (Section 3). The results obtained are presented and discussed in Section 4 and conclusions are summarized in Section 5.

# 2 Methods

This section begins with a short overview of the modeling framework, followed by a description of the methods used to estimate the model parameters and to assess the model performance.

## 2.1 Model framework

The analytical model framework for probabilistic characterization of rainfall-driven daily discharges developed by Botter et al. (2007c) is based on a previous model proposed by Rodriguez-Iturbe et al. (1999). This original model represents the dynamics of soil moisture at a point as a result of a deterministic state-dependent loss function, combined with stochastic increments triggered by rainfall events. Accordingly, Botter et al. (2007c) proposed to describe the dynamics of daily stream flow considering that some precipitation events act as a stochastic forcing for discharge production and that water is released from the soil producing discharge according to a deterministic recession.

It is assumed hereby that the discharges, $Q$, are the result of a sequence of subsurface inputs triggered by precipitation events that deliver enough water to fill the water deficit in the soil and to raise its level of moisture above its retention capacity. The excess of water becomes discharge and is removed from the soil as subsurface run-off. Originally, the subsurface storage was assumed to behave like a linear reservoir with a constant $k_l$. The overall rainfall forcing can be modeled as a marked Poisson process with frequency $\lambda_P$ and exponentially distributed rainfall depths with average $\alpha$. But not all the rainfall events produce discharge because of the losses due to evapotranspiration and retention in the soil. Accordingly, $\lambda_P$ is reduced to $\lambda$: the frequency of discharge-producing events, i.e., of events that raise the soil moisture above its retention capacity. In rainfall-driven environments, the reduced frequency

$\lambda$ can be understood as the frequency of the rainfall events that are unusable by the plants; is influenced by the soil storage capacity and soil drying time (Botter et al., 2007b). From those assumptions, it is possible to obtain the following probabilistic distribution for daily discharges that has the shape of a gamma distribution:

$$p(Q, t \to \infty) = \frac{1}{\Gamma\left(\frac{\lambda}{k_l}\right)} \frac{1}{Q} \left(\frac{Q}{\alpha k_l A}\right)^{\frac{\lambda}{k_l}} \exp\left(-\frac{Q}{\alpha k_l A}\right) \tag{1}$$

where $A$ is the catchment area and $\Gamma$, the gamma function. Botter (Botter et al., 2009) extended this framework to consider a nonlinear recession and obtained a new equation to describe daily discharges:

$$p(Q, t \to \infty) = C \left\{ \frac{1}{Q^a} \exp\left[ -\frac{Q^{2-a}}{\alpha k_n (2-a)} + \frac{Q^{1-a}\lambda}{k_n(1-a)} \right] \right\} \tag{2}$$

where $a$ and $k_n$ are the nonlinear recession coefficients (for a recession represented by $dQ/dt = k_n Q^a$) and $C$ is a normalizing constant (Botter et al., 2009).

## 2.2 Time intervals

The model is suitable for steady state conditions, at the annual or seasonal scales, depending on the temporal variability of the model parameters (Botter et al., 2007a). The most common periods of application are the four meteorological seasons (Botter et al., 2007c, 2008, 2013; Ceola et al., 2010), but can be different from those.

In the selected study of Switzerland, there is a wide variety of hydrological regimes, that can be classified in pluvial, snow-dominated and glacier. In pluvial regimes, the trigger to discharge production is rainfall;;2 and in glacier regimes there is an inter-annual accumulation of snow. In general, precipitation is well distributed along the year, but there is a strong seasonality due to snow processes that affect the values of the model parameters. Taking into account that seasonality, the model framework is applied to five different time intervals: the civil year (01-Jan to 31-Dec) and the meteorological seasons (spring from 01-Mar to 31-May, summer from 01-Jun to 31-Aug, autumn from 01-Sep to 30-Nov and winter from 01-Dec to 28-Feb).

## 2.3 Parameter estimation

The model parameters are related to the stochastic inputs or to the deterministic recession. For each time interval, all the parameters are firstly calculated in a forward mode, i.e., directly from the data, without calibration. Additionally, the recession parameters are also calibrated to optimize the results. The calibration is performed by fixing the stochastic inputs parameters and optimizing the recession parameters using maximum likelihood estimates.

Stochastic inputs parameters are the mean depth of precipitation, $\alpha$, and the frequency of the events that produce discharge, $\lambda$. $\alpha$ can be obtained as the mean of the positive daily effective precipitation after subtracting interception losses from the observed daily precipitation depths. Different maximum interception depths were adopted according to land cover (4 mm for forests, 2 mm, for low vegetation, 1 mm for impervious areas, 0 mm for water bodies (Gerrits, 2010)). The catchment-scale maximum interception depth is obtained as the area-average of these values but a minimum interception depth of 1 mm is imposed. This catchment-scale interception depth is subtracted from daily precipitation observations, assuming that at a daily time step, all intercepted water evaporates (Santos et al., 2018a). $\lambda$ is obtained from a combination of the remaining daily precipitation data and the equivalent daily discharges from the relation $\overline{Q} = \lambda\alpha$ where $\overline{Q}$ is the long term average of the observed daily discharges, $Q$, in the time interval being considered. The estimation based on this method has been shown to provide the best results (Ceola et al., 2010), and it was used by the majority of studies since then (Basso et al., 2015; Botter et al., 2013; Ceola et al., 2010). Deterministic recession parameters are obtained by means of recession analysis, which comprehends two steps: recession extraction and parameter estimation. Recession extraction refers to the selection of discharge data in periods when the only source of stream flow is the water stored in the soil. Such data will be used in the parameter estimation step.

Hereby, the method applied for recession extraction followed the work of (Dralle et al., 2017), who suggest that recessions should be selected based on an upward concavity requirement in the hydrographs, with a minimum length of four days. Those authors also studied the influence of the peak selectivity criterion, concluding that it does not interfere significantly in the results. So, we have chosen a simple method, selecting only recessions that begin with a discharge higher than the annual or seasonal long term mean discharge, depending on the time interval under analysis. This type of criterion has been adopted previously by other authors (Biswal and Marani, 2010; Mutzner et al., 2013). The chosen extraction method works well with this model, but is very restrictive in periods with very low discharges, such as autumn and winter in glacier catchments. So for those catchments the selection was less strict and did not follow the peak selectivity criterion.

Parameter estimation is based on the linear least-squares method that fits a recession parameter set ($k_{ni}$ and $a_i$) to each selected recession event linearized by a $\log - \log$ scale. The exponent $a$ is then taken as the median value of the fitted $a_i$ values, where $i$ is an index for the individual recessions. Once $a$ is fixed, the curves are fitted again to estimate the coefficient $k_n$ as the median of the recalculated $k_{ni}$ (Basso et al., 2015; Mutzner et al., 2013). For the linear recession, the recessions are fitted considering a fixed exponent $a = 1$ - (Eq. 2) - and the median is taken as the parameter.

## 2.4 Performance evaluation

Taking into account that the FDC can be understood as a probabilistic distribution of daily discharges that can also be represented as a cumulative distribution function (cdf), the Kolmogorov-Smirnov distance ($c^{KS}$) is used to assess the performance of the model, as previously done (Ceola et al., 2010; Santos et al., 2018a). The $c^{KS}$ represents the maximum distance or probability gap between an analytical cdf derived from the model and an empirical cdf.

# 3 Cases

The three Swiss catchments adopted as case studies are those at the river gauges stations of Murg at Wangi (MUW), with a pluvial regime, Grosstalbach (GRO), with a snow-dominated regime, and Rhone at Gletsch (RHG), with a glacier regime, the locations of those catchments are shown in Fig. 1.

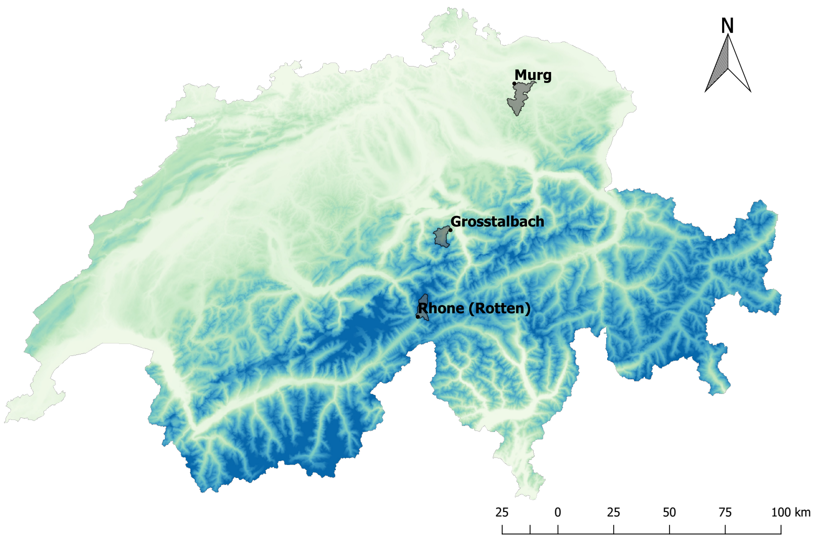


Figure 1: Localization of the 3 case studies in Switzerland over a simplified topographic map

Daily discharge data for each catchment were provided by the Swiss Federal Office for the Environment (FOEN) (FOEN, 2017) and daily precipitation data was extracted from a gridded database (MeteoSwiss, 2014). Since precipitation is gridded and the model is lumped, we adopted an spatial average of the data from cells corresponding to the selected catchments. For additional information about the catchments, such as land use, required to calculate interception, there is a dataset associated to the catchments (Aschwanden, 1996). Table 1 shows some key characteristics of the catchments selected as case studies.

Table 1: Characteristics of case studies

| ID | Name | Area ($km^2$) | Mean elevation (masl) | Station elevation (masl) | Regime |
|---|---|---|---|---|---|
| 2126 | Murg - Wängi | 78.9 | 650 | 466 | pluvial |
| 2276 | Grosstalbach | 43.9 | 1820 | 767 | snow-dominated |
| 2268 | Rhone - Gletsch | 38.9 | 2719 | 1761 | glacier |

# 4 Results and discussion

The values of the parameters and of the performance indicator $c^{KS}$ for the three case studies are shown in the Tables 2 and 3. When analyzing the tables one should have in mind that the best values of the parameters are those obtained by inverse estimation. The parameters that belong to both linear and nonlinear models (common parameters) are shown in Table 2. The cdfs derived from the FDCs for the linear and nonlinear models are presented in Figures 2 to 7. The shaded area in the figures represents the natural variability of the daily streamflows.

Table 2: Mean daily streamflows, $\overline{Q}$ and common model parameters. For the linear model, recession parameters and $c^{KS}$ values for each season. The lower indexes $lf$, $li$, stand for linear forward and linear inverse, respectively.

| Catchment and period | Common parameters | | | | Linear model | | | |
|---|---|---|---|---|---|---|---|---|
| | $\overline{Q}$ (mm) [1] | $\alpha(mm)$ [2] | $\lambda_P$ [3] | $\lambda$ [4] | $k_{lf}$ [5] | $c_{lf}^{KS}$ [6] | $k_{li}$ [7] | $c_{li}^{KS}$ [8] |
| MUW | | | | | | | | |
| Summer | 1,66 | 9,58 | 0,35 | 0,17 | 0,37 | 0,36 | 0,10 | 0,12 |
| Autumn | 1,63 | 8,59 | 0,30 | 0,19 | 0,39 | 0,36 | 0,12 | 0,13 |
| Winter | 2,47 | 6,39 | 0,40 | 0,39 | 0,27 | 0,10 | 0,22 | 0,09 |
| Spring | 2,37 | 7,39 | 0,36 | 0,32 | 0,25 | 0,15 | 0,15 | 0,10 |
| Year | 2,03 | 8,06 | 0,35 | 0,25 | 0,31 | 0,22 | 0,15 | 0,10 |
| GRO | | | | | | | | |
| Summer | 5,97 | 11,87 | 0,44 | 0,50 | 0,18 | 0,12 | 0,11 | 0,05 |
| Autumn | 2,61 | 10,69 | 0,33 | 0,24 | 0,22 | 0,24 | 0,08 | 0,09 |
| Winter | 1,28 | 8,97 | 0,39 | 0,14 | 0,33 | 0,50 | 0,04 | 0,11 |
| Spring | 4,17 | 9,41 | 0,42 | 0,44 | 0,17 | 0,13 | 0,24 | 0,06 |
| Year | 3,54 | 10,32 | 0,40 | 0,34 | 0,20 | 0,09 | 0,20 | 0,09 |
| RHG | | | | | | | | |
| Summer | 17,12 | 9,01 | 0,51 | 1,90 | 0,20 | 0,12 | 0,42 | 0,06 |
| Autumn | 4,97 | 12,08 | 0,41 | 0,41 | 0,20 | 0,20 | 0,32 | 0,11 |
| Winter | 0,67 | 12,05 | 0,44 | 0,06 | 0,04 | 0,26 | 0,01 | 0,04 |
| Spring | 1,85 | 10,72 | 0,50 | 0,17 | 0,14 | 0,14 | 0,15 | 0,14 |
| Year | 6,21 | 10,88 | 0,47 | 0,57 | 0,18 | 0,46 | 0,87 | 0,17 |

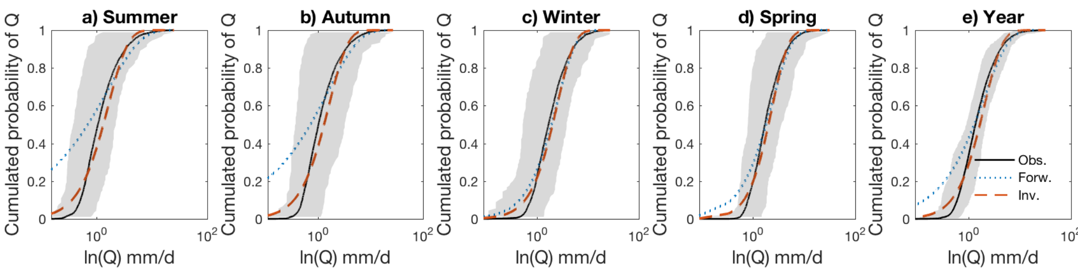


Figure 2: Cdfs for the linear model in MUW

The analysis of the results begun by the common parameters of Table 2, that are directly related to the characteristics of discharge and precipitation in the catchments. It is possible to notice that the intra-annual variability in the streamflows (col. [1]) increases as there are more snow processes. In the snow-dominated catchment (GRO), streamflows decrease during autumn and winter, due to snow accumulation and then,

Table 3: For the nonlinear model, recession parameters and $c^{KS}$ values for each season. The lower indexes $nf$, $ni$, stand for nonlinear forward and nonlinear inverse, respectively.

| Catchment and period | Nonlinear model | | | | | |
|---|---|---|---|---|---|---|
| | $k_{nf}$ [9] | $a_f$ [10] | $c^{KS}_{nl}$ [11] | $k_{ni}$ [12] | $a_i$ [13] | $c^{KS}_{ni}$ [14] |
| MUW | | | | | | |
| Summer | 0,05 | 2,26 | 0,08 | 0,08 | 1,90 | 0,01 |
| Autumn | 0,11 | 1,93 | 0,10 | 0,08 | 1,98 | 0,02 |
| Winter | 0,09 | 2,16 | 0,03 | 0,09 | 2,08 | 0,02 |
| Spring | 0,04 | 2,24 | 0,20 | 0,14 | 1,75 | 0,02 |
| Year | 0,07 | 2,17 | 0,09 | 0,11 | 1,83 | 0,01 |
| GRO | | | | | | |
| Summer | 0,01 | 2,80 | 0,29 | 0,18 | 1,32 | 0,04 |
| Autumn | 0,00 | 3,13 | 0,13 | 0,02 | 1,86 | 0,02 |
| Winter | 0,03 | 2,39 | 0,05 | 0,03 | 2,07 | 0,03 |
| Spring | 0,06 | 2,17 | 0,16 | 0,03 | 2,61 | 0,02 |
| Year | 0,01 | 2,77 | 0,32 | 0,12 | 1,57 | 0,05 |
| RHG | | | | | | |
| Summer | 0,02 | 3,32 | 0,37 | 0,13 | 1,85 | 0,05 |
| Autumn | 0,00 | 2,65 | 0,15 | 0,90 | 0,70 | 0,04 |
| Winter | 0,01 | 3,06 | 0,30 | 0,15 | 1,67 | 0,07 |
| Spring | 0,62 | 6,58 | 0,51 | 0,02 | 1,62 | 0,02 |
| Year | 0,01 | 3,06 | 0,51 | 0,56 | 1,54 | 0,10 |

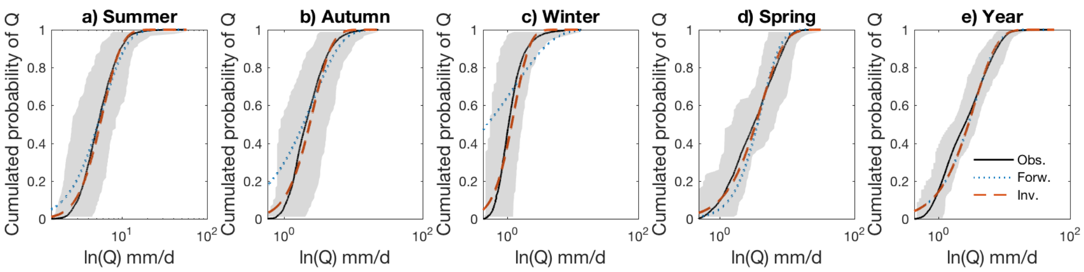


Figure 3: Cdfs for the linear model in GRO

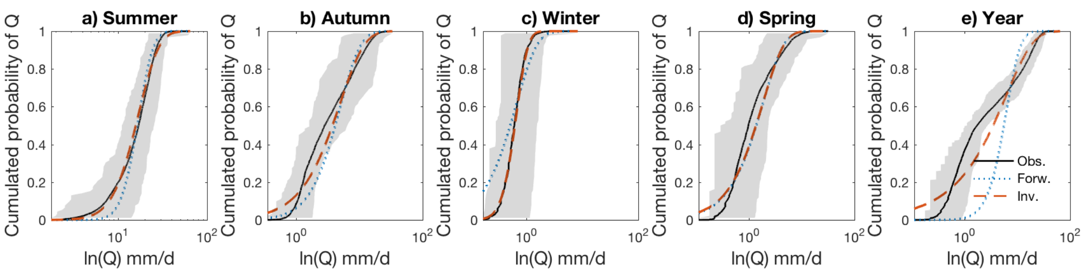


Figure 4: Cdfs for the linear model in RHG

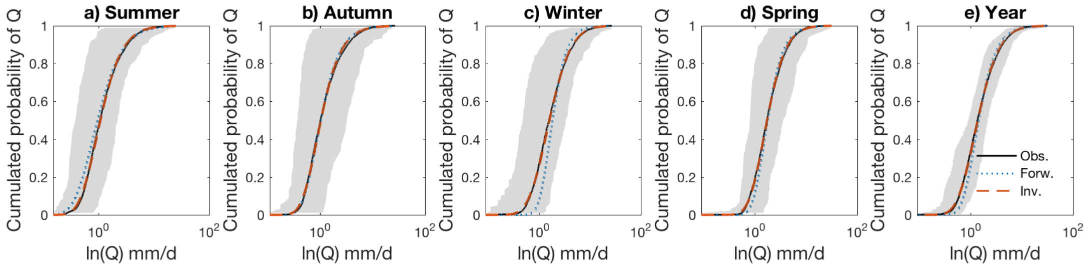


Figure 5: Cdfs for the nonlinear model in MUW

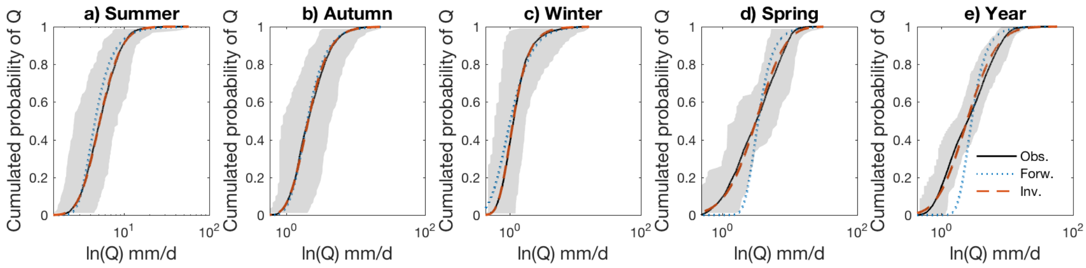


Figure 6: Cdfs for the nonlinear model in GRO

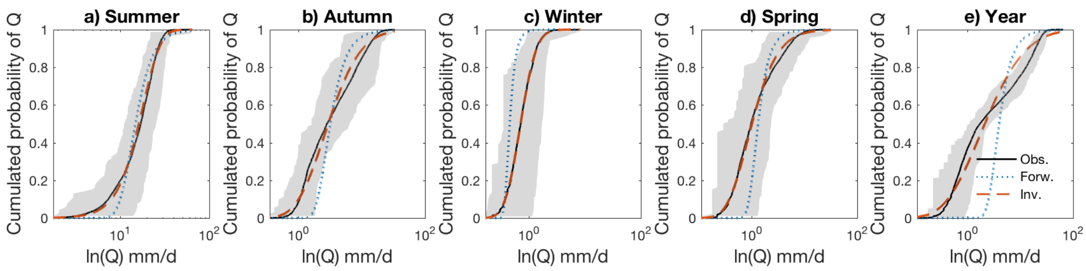


Figure 7: Cdfs for the nonlinear model in RHG

during spring and summer they raise, due to snow melt. The seasonal variability is more pronounced in the glacier catchment (RHG). Nevertheless, it is interesting to notice that the parameters related to the precipitation (i.e $\alpha$, column [2] and $\lambda$, column [3]) do not follow the same trends, showing that the variability in the streamflow does not happen as a consequence of a variability in the precipitation.

By comparing the frequency of the precipitation events, $\lambda_P$ (col. [3], Tab. 2), and of the discharge producing events, $\lambda$ (col. [4], Tab.2), it is also possible to notice some patterns. In theory, the values of $\lambda$, should be smaller than those of $\lambda_P$ because of the water losses, mostly by evapotranspiration. This can be seen in the pluvial catchment (MUW), in which the highest difference between the previous frequencies happen during summer, the warmer season with higher evapotranspiration. But for the snow-dominated and glacier catchments (GRO and RHG), that does not always happen, $\lambda$ being higher than $\lambda_P$ during summer and, in the former catchment, in spring and much smaller during winter. The increase of $\lambda$ during summer, which was previously observed by Santos et al. (2018a), combined with its decrease during winter show that the frequency parameter is able not only to incorporate the additional water supply from snow-melt, but also the effect of snow accumulation.

Regarding the recession parameters of the linear model (Figures 2 to 4), the comparison between the forward (col. [5], Tab. 2) and the inverse (col. [7], Tab. 2) estimates shows that, in general, for pluvial and snow-dominated conditions, the forward method tends to overestimate the parameters, while for glacier catchment the parameters are underestimated, especially in summer when there is a strong contribution of the snow-melt. The remarkably low values of the parameters $k_{li}$ for winter in the snow-dominated and glacier catchments are coherent with the work of (Schaefli et al., 2013), that proposed to incorporate the effect of snow accumulation in the model by adding a delay in the response time of that catchment, that delay being translated as a decrease in the linear recession parameter. Observing col. [8] of Table 2, it is noticeable that, in general, the linear model works better for catchments with snow processes (smaller $c^{KS}$) and the seasonal performances tend to be better than the annual ones for the same catchments. The performances for summer and spring for the snow-dominated catchment are remarkably good as are the summer and winter performances for the glacier catchment.

Regarding the nonlinear model (Figures 5 to 7), its performance (col. [11] and [14] of Table 3) is generally better than that of the linear model (col. [6] and [8] of Table 2), especially for the pluvial catchment. Again, the performance of the model is obviously much better for the inverse estimation.

Finally, a very important observation in terms of future development is that the seasonal performances for both models in catchments with snow-processes are better than the annual ones. The shape of the empirical cdfs in Figures 4e) and 7e) elucidates the poor annual performance. This indicates that future efforts should focus on obtaining and combining seasonal curves instead of considering single annual ones.

# 5 Conclusions

The following conclusions are worth emphasizing:

- The performance of the nonlinear model for pluvial catchments is very good, either on a seasonal or yearly basis, showing that this model is able to represent cdfs in those catchments without requiring developments.
- The seasonal performance of the linear model is better for the catchments with snow-processes (snow-dominated and glacier) than it is for the pluvial catchment.
- The annual performace of the model for glacier catchments is poor. That happens because of the different shape of the empirical cdf and new developments are required so the analytical model will be suitable also for glacier catchments.
- For inverse estimations, both linear and nonlinear models have annual performances for the snow-dominated but mainly for the glacier catchments that are poorer than the seasonal ones which indicates that further developments aiming at obtaining annual cdfs should be focused on the improvement and merging of the seasonal curves.

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