Example Problem for a Concentrated Load

Consider the same deck as in the previous example (page 31). This problem is designed to demonstrate how to check the ability of a composite slab to carry a 3000 lb point load over an area of 4.5" x 4.5" occurring anywhere in the span. This problem is consistent with the requirements of the 2006 International Building Code for garages storing vehicles accommodating not more than nine passengers.

There will be no other live load acting simultaneously, and there is no negative bending reinforcement present over the supports, therefore we assume a single span condition.

For this example the following criteria apply:
Clear Span - 9 ft. -- Maximum Unshored Span is 9.67 ft
Slab Thickness - 4.5 in.
Composite Properties:

ϕM_{no} - 42.94 in.k
ϕM_{nf} - 57.78 in.k
w_{deck} - 1.8 psf
w_{slab} - 42 psf
I_{av} - 6.3 in^4/ft
ϕV_{nt} - 5970 lbs.
$b_2 = b_3 = 4.5$ in.
$b_m = b_2 + 2t_c + 2t_t$
t_c = Thickness of concrete cover over the top of the deck
t_t = Thickness of any additional topping
h = Total thickness exclusive of topping
t_t = 0 in.
t_c = 2.5 in.
h = 4.5 in.

$b_m = 4.5 + 2(2.5) + 0 = 9.5$ in.

For moment and for determining the distribution steel, put the load in the center of the span.
$b_e = b_m + 2(1\text{-}x/l)x$; where x is the location of the load: x = l/2
$b_e = 9.5 + 2(1\text{-}54/108)54 = 63.5$ in.; However b_e 8.9(t_c/h) in feet
8.9(2.5/4.5)(12) = 59 in. therefore b_e = 59 in.

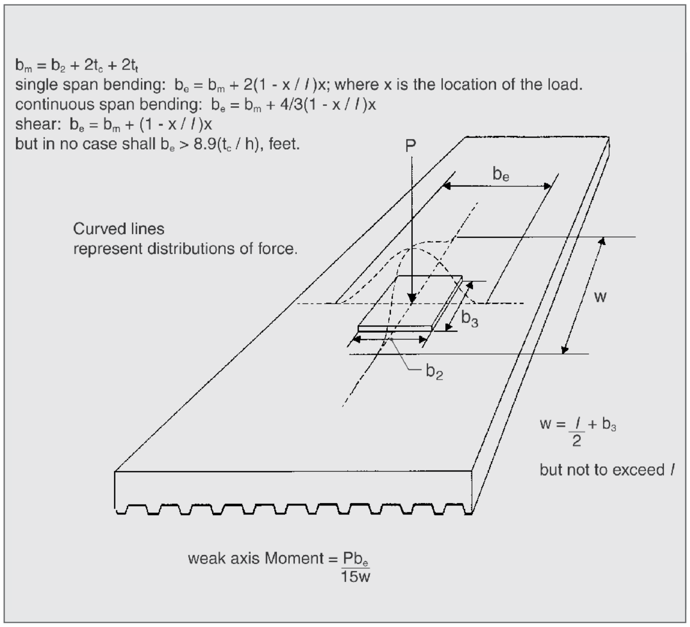


FIGURE 1

Check vertical shear :
Put the load one slab depth away from the beam; x = h
$b_{ve} = b_m + (1\text{-}x/l)x = 9.5 + (1\text{-}4.5/108)4.5 = 13.8$ 59

For Moment b_e = 59 in.
For Shear b_{ve} = 13.8 in.

Live load moment (per foot of width) = Pl/4
= (1.6)(3000)(9/4)(12/59)(12)/1000

Pl/4 = 26.36 in.k;
1.6 is the load factor and 12/59 is the distribution factor

w_{total} = Total dead load = $w_{slab} + w_{deck} = 42 + 1.8 = 43.8 \cong 44$ psf;
1.2 is the load factor.

Dead load moment = $w_{total}l^2/8 = 1.2(44)(9)^2(12)/8000 = 6.42$ in.k

26.36 + 6.42 = 32.78 in.k

ϕM_{no} ; Factored resisting moment when studs are not present on the beams

ϕM_{no} = 42.94 in.k > 32.78 in.k O.K.

V = 1.6(3000)(12/13.8) + 1.2(44)(9)/2 = 4412 lbs

ϕV_{nt} = 5970 lbs > 4412 lbs O.K.

Find the required distribution steel (welded wire mesh)

M_2 = Weak direction moment = $Pb_e/15W$

W = l /2 + b_3 = 54 + 4.5 = 58.5 in. 108 in.

M_2 = 1.6(3000)(59)(12)/(15 x 58.5) = 3873 in.lbs/ft

Assume the wire mesh is located 1/2" above top of deck;
d = 4.5 - 2 - 0.5 = 2 in.

$M_n = A_sF_y(2\text{-}a/2)$
A_s is the area per foot of the wire mesh which has an F_y of 60 ksi.
If the bars are being investigated, the F_y would have to be ad usted accordingly.

$a = A_sF_y/0.85f'_cb$; b = 12 in.
Assume A_s is the area of 6 x 6 - W2.0 x W2.0 mesh. A_s= 0.04in^2/ft.
6 x 6 - W1.4 x W1.4 mesh is the ANSI/SDI minimum.

a = 0.040(60000)/(0.85 x 3000 x 12) = 0.078 in.

NOTE: ϕ = 0.9 in ACI but SDI uses 0.85

ϕM_{weak} = 0.85(0.040)(60000)(2-0.078/2) = 4000 in.lbs/ft

4000 > 3873 O.K. 6 x 6 - W2.0 x W2.0 mesh is sufficient.

continued on next page

FLOOR DECK

EXAMPLE PROBLEM

Composite Floor Deck Design - Example Problem

For the composite floor deck, composite section properties and concentrated load example problems, the following data from the Canam *Steel Decks for Floors and Roofs* manual will be used and are worked using LRFD methodology.

Deck Type – **2" LOK-FLOOR COMPOSITE**
Gage - **20 (t=0.0358")**
Yield Stress – **40 KSI (MINIMUM)**
The deck properties (per foot of width) have been calculated in accordance with the AMERICAN IRON AND STEEL INSTITUTE (AISI) SPECIFICATIONS and are:
I_p= 0.390 in.^{4}, (Moment of inertia in positive bending);
S_p= 0.332 in.^{3}, (Section modulus in positive bending);
S_n= 0.345 in.^{3}, (Section modulus in negative bending);
A_s=0.54 in.^{2}; (Steel deck area per unit width);
φR_{bi}= 1360 lbs. (Factored web crippling capacity based on 5" interior bearing;
φR_{be} = 800 lbs. (Factored web crippling capacity based on 2.5" exterior bearing;
φV_n= 2930 lbs. (Factored deck shear strength);
w = 1.8 psf. (Deck weight)
SDI tolerances apply.
The concrete properties are:
f_c= 3 ksi (Concrete strength);
γ = 145 pcf (Concrete density);
Modular ratio, **n^* = E_s/E_c = 9**

*The current AISC/SDI method of calculating E_c is:

$E_v = \gamma^{1.5}\sqrt{f_c}, ksi = 145^{1.5}\sqrt{3.0}x1000 = 3.024x10^6\,psi$ E_s =29.5 x 10^6 psi,

So modular ratio **n** = 9.75.
However, for the sake of consistency with the composite deck tables, we will use the **n** historically used by SDI, which is 9 and gives an E_c = 3.28x10^6 psi.

Unshored Span Calculation

Calculate the maximum unshored clear span for the three span condition of the deck with a 4.5" slab. The resistance factors are provided by the AISI Specifications. The load factors are **1.6 for concrete weight, 1.4 for construction loading of men and equipment, and 1.2 for the deck dead load.** It is important to remember that these factors are for the deck under the concrete placement loads; when the slab has cured, and the system is composite, the factors are different.

REFER TO PAGE 30 FOR FIGURE SHOWING 3 SPAN CONDITION.

W1 = $w_{concrete}$ + w_{deck} = 42 psf + 1.8 psf; W2 = construction load = 20 psf.

φ = 0.95 and is the AISI resistance factor using the deck as a form,
F_y = yield stress of steel deck ≤ 60 ksi

Bending is checked using the controlling sequential loading.

Check negative bending with two spans loaded: where l = span, applied bending moment $-M = 0.117(W1+W2)l^2$ and deck resistance $= \dfrac{\phi F_y S_n}{12}$

Solving for length:

$$0.117\,l^2(1.6x42 + 1.4x20 + 1.2x1.8) = \frac{0.95(40000)(0.345)}{12};\ l = 9.79'$$

Now check positive bending with one span loaded with concrete and the concentrated load:

$+M = 0.20Pl + 0.094W1l^2$, Deck Resistance $= \dfrac{\phi F_y S_p}{12}$

$$0.20(1.4x150)l + 0.094(1.6x42 + 1.2x1.8)l^2 = \frac{0.95(40000)(0.332)}{12};\ l = 9.88'$$

Check positive bending with one span loaded with concrete and construction load:

$+M = 0.094(W1+W2)l^2$, Deck Resistance $= \dfrac{\phi F_y S_p}{12}$

$$0.094(1.6x42 + 1.2x1.8 + 1.4x20)l^2 = 1051.33;\ l = 10.72'$$

Web crippling, shear, and the interaction of bending and shear are checked with two spans loaded. Check interior web crippling with:

$\phi R_{bi} = 1.2(W1)l + P$ or $\phi R_{bi} = 1.2(W1+W2)l$

$\phi R_{bi} = 1.2(1.6x42 + 1.2x1.8)l + 1.4\ x\ 150 = 1360;\ l = 13.82'$ or

$\phi R_{bi} = 1.2(1.6x42 + 1.2x1.8 + 1.4x20)l = 1360;\ l = 11.64'$

Check exterior web crippling:

$\phi R_{be} = 0.433(W1)l + P$ or $\phi R_{be} = 0.4(W1+W2)l$

$\phi R_{be} = 0.433(1.6x42 + 1.2x1.8)l + 1.4\ x\ 150 = 800;\ l = 19.65'$ or

$\phi R_{be} = 0.4(1.6x42 + 1.2x1.8 + 1.4x20)l = 800;\ l = 20.54'$

Shear or bending alone will not control, but the interaction of shear and bending could. The AISI equation C3.3.2-1 for interaction is:

$$\sqrt{\left(\frac{\overline{M}}{\phi_b M_n}\right)^2 + \left(\frac{\overline{V}}{\phi_v V_n}\right)^2} \le 1.0$$

$-M = 0.117(W1+W2)l^2 = .117l^2(1.6x42 + 1.4x20 + 1.2x1.8)x12 = 136.69l^2$

$\phi M_n = 0.95F_y S_n = 0.95x40000x.345 = 13110\ in-lbs.$

$V_{applied} = 0.617(W1+W2)l = 0.617(1.6x42 + 1.4x20 + 1.2x1.8)l = 60.07l$

$$\sqrt{\left(\frac{136.69l^2}{13110}\right)^2 + \left(\frac{60.07l}{2930}\right)^2} \le 1.0 \qquad l = 9.69'$$

Check deflection with $\delta = l/180$ and with $\delta \le 0.75''$

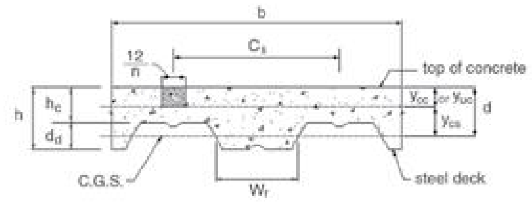
$$\delta = \frac{0.0069(W_1)l^4}{EI} \qquad \delta = \frac{.0069(42+1.8)l^4x1728}{29.5x10^6x.390} = \frac{lx12}{180} \qquad l = 11.36'$$

$$\delta = \frac{.0069(42+1.8)l^4x1728}{29.5x10^6x.390} \le 0.75'' \qquad l = 11.34'$$

Combined bending shear governs, tables show a maximum unshored span of 9.67' and agrees well with 9.69'.

Composite Section Properties

Calculate the composite section properties and the allowable uniform load for the deck combination. The clear span is 9'. No negative bending reinforcing is used over the beams, so the composite slab will be a simple span.
Determine the "cracked" moment of inertia (I_{cr}). This calculation is the standard ASD calculation which assumes all concrete below the neutral axis is cracked. The concrete is transformed into equivalent steel.



When y_{cc} is equal to or less than the depth of concrete above the top of steel, h_c, that is, $y_{cc} \le h_c$, then

$y_{cc} = d\left\{\sqrt{2\rho n + (\rho n)^2} - \rho n\right\}$ where $\rho = \dfrac{A_s}{bd} = \dfrac{0.54}{12(4.5-1)} = 0.0129$

$y_{cc} = 3.5\left\{\sqrt{2(.0129x9) + (.0129x9)^2} - .0129x9\right\} = 1.328 \approx 1.33'' < 2.5''\,O.K.$

Determine I_{cr}: where $y_{cs} = d - y_{cc} = 3.5 - 1.33 = 2.17''$

$$I_{cr} = \frac{b}{3n}y_{cc}^3 + A_s y_{cs}^2 + I_{sf} \qquad I_{cr} = \frac{12''}{3x9}1.33^3 + 0.54x2.17^2 + 0.39 = 3.98\ in.^4$$

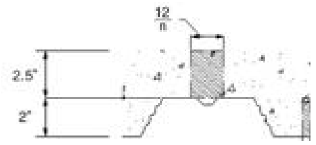
Determine cracked section modulus: $S_c = \dfrac{I_{cr}}{h - y_{cc}} = \dfrac{3.98}{4.5-1.33} = 1.25\ in.^3$

Table's print out shows 1.25 which checks.

FLOOR DECK

EXAMPLE PROBLEM

Determine un-cracked moment of inertia (I_u). The concrete is again transformed into equivalent steel.



Using the top of the slab as the reference line:

$$y_{uc} = \frac{0.5bh_c^2 + nA_s d + W_r d_d (h - 0.5d_d)\frac{b}{C_s}}{bh_c + nA_s + W_r d_d \frac{b}{C_s}}$$

$$= \frac{0.5x12x2.5^2 + 9x0.54x3.5 + 6x2(4.5 - 0.5x2)^{12}/_{12}}{12x2.5 + 9x0.54 + 6x2x^{12}/_{12}} = 2.06 in.$$

and $y_{cs} = d - y_{uc}$ $\quad y_{cs} = 3.5 - 2.06 = 1.44"$

and the uncracked I is:

$$I_u = \frac{bh_c^3}{12n} + \frac{bh_c}{n}(y_{uc} - 0.5h_c)^2 + I_{sf} + A_s y_{cs}^2 + \frac{W_r b d_d}{nC_s}\left[\frac{d_d^2}{12} + (h - y_{uc} - 0.5d_d)^2\right]$$

$$I_u = \frac{12x2.5^3}{12x9} + \frac{12x2.5}{9}(2.06 - 0.5x2.5)^2 + 0.39 + 0.54x1.44^2$$

$$+ \frac{6x12x2}{9x12}\left[\frac{2^2}{12} + (4.5 - 2.06 - 0.5x2)^2\right] = 8.64 in.^4$$

Moment of inertia of composite section considered effective for deflection computations is:

$I_{av} = \frac{I_u + I_{cr}}{2}$ (Transformed to steel) $\quad I_{av} = \frac{8.64 + 3.98}{2} = 6.3 in.^4$ This agrees with tables.

Calculate the live load allowed for the case with no studs with clear span = 9 feet. The nominal resisting moments where the φ factor is 0.85 is:
$\phi M_{no} = \phi F_y S_e = 0.85x40000x1.25 = 42500 in - lbs.$ The table shows 42.94 in.-k which checks within 1%.

Unless negative bending is present, the composite slab is assumed to be single span. For single span, the un-factored uniform (live) load (W_L) is found by:

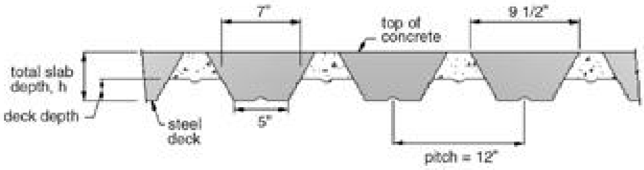
$\phi M_o = \frac{(1.6W_L + 1.2W_D)l^2 x12}{8}$ $\quad W_D$=dead load=42+1.8=43.8 psf. Solve for W_L.

$$W_L = \frac{\frac{8x42500 in. - lbs./ft.}{9^2 ft.^2 x12 in./ft.} - 1.2x43.8 psf}{1.6} = 186 psf$$

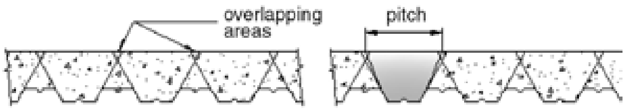
This is within 2% of the 190 psf value published in the table.
Check deflection if applied load is 190 psf. SDI maximum deflection is 0.75" or L/180.

$\delta = \frac{0.013Wl^4}{EI_{avg}} = \frac{0.013x190x9^4(1728)}{29.5x10^6 x6.3} = 0.15" < 0.75" and < \frac{l}{180}$ which is O.K.

Check the factored vertical shear capacity: $\phi V_{steel\ deck}$=2930 pounds (per foot of width). Where A_c is the concrete trapezoidal (shaded) area available to resist shear = 32.6 in^2 from table.



If the slab depth causes the area to overlap, then the area is adjusted to not exceed the shape provided with the deck pitch as the top dimension.



$\phi V_{concrete} = \phi 2(f_c')^{1/2} A_c = .85x2x3000^{1/2}32.6 = 3035 lbs.$
The factored shear resistance of the composite slab
V_{nt} = 2930 + 3035 = 5965 ≈ 5970 lbs.

The total of the φ $V_{steel\ deck}$ and the φ $V_{concrete}$ is not allowed to exceed the concrete shear control limit of:
$\phi 4(f_c')^{1/2} A_c = .85x4x3000^{1/2}32.6 = 6071 lbs.$ > the tabulated 5970 lbs. O.K.
Note: If the concrete density ≤ 130 pcf V is multiplied by 0.75.

The un-factored live load allowed if shear controls (W_v) is found by:
$5965 = \frac{(1.6W_v + 1.2W_d)l}{2}$ $\quad$ Solve for W_v.

$$W_v = \frac{\frac{5965x2}{l} - 1.2W_d}{1.6} = \frac{\frac{5965x2}{9} - 1.2x43.8}{1.6} = 796 psf$$ < 190 psf

so shear does not control. O.K.

The number of studs required to develop 100% of the factored moment is given by:

$N_s = \frac{F_y(A_s - A_{webs}/2 - A_{bf})}{0.221(f_c' E_c)^{1/2}}$ $\quad N_s = \frac{40(.54 - 0.16/2 - 0.179)}{0.221(3000x3.28x10^6)^{1/2} \div 1000} = 0.51$

studs per foot, which checks the table.
The inverse 1.0/0.51 = 1.95 which means a stud is required every 1.95 feet in order to achieve the full factored moment.

The nominal (ultimate) moment capacity with studs on beam:
$\phi M_{nf} = \phi A_s F_y (d - a/2)$ where (a) is the depth of the concrete compression block and is given by: $a = \frac{A_s F_y}{0.85 f_c' b}$

where (b) is the unit width of 12". $\quad a = \frac{0.54x40000}{0.85x3000x12} = 0.71"$

$\phi M_{nf} = 0.85x0.54x40000(3.5 - 0.71/2) = 57780 in. - lbs.$
The tables show 57.78 in-k which checks.

Since N_s = 0.51 and 1/N_s = 1.95', studs spaced at 1' will develop the full factored moment of 57.78 in-k, and with no studs the composite slab develops 42.94 in-k. When the shear studs are present on the beam supporting the composite steel deck, but are not present in sufficient quantity to develop the ultimate capacity of the section in bending then the composite slab capacity is found by interpolation. If studs are spaced at 3'(1/3=0.33 studs per foot) then the usable nominal moment capacity at stud density N'_s is:

$$\phi M_{np} = M_{no} + (M_{nf} - M_{no})\frac{N'_s}{N_s} \le M_{nf}$$

$$\phi M_{np} = 42.94 + (57.78 - 42.94)\frac{0.33}{0.51} = 52.49 in - k \le 57.78 in - k$$