SAFETY IN NUMBERS?

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ABSTRACT

Using numbers to evaluate dam safety is ingrained in our current engineering practice. This paper describes why numbers are necessary but not sufficient in evaluating safety and in conveying risk to the public we serve. Therefore, engineering judgment is advocated to test the reasonableness of numbers and to address safety issues not amenable to rational calculation such as the likelihood of human error leading to a loss of reservoir control.

Engineering judgment is an acquired skill, born of experience. It is a way of thinking anchored in common sense. It may or may not be informed by numbers. It benefits from visual observation and is likely to use words instead of numbers to express probabilities. It recognizes that predicting human behavior is not possible, and it seeks to avoid human error by training and practice. Engineering judgment provides a platform from which the reasonableness of safety estimates may be assessed.

HISTORY

How did we get to our current reliance on numbers? And is there really safety in numbers?

"I have great respect for the past. If you don't know where you've come from, you don't know where you're going." Maya Angelou

For more than 5,000 years, our ancestors constructed dams to store and divert water for irrigation and domestic use. Dams were conceived without numbers as we know them. It was the Age of Empiricism when folks built, learned from what did and didn't work, and then modified their designs



modified their designs Figure 1 – Proserpina Dam Figure 2 – Kavir Dam until stable structures were achieved. Ancient dams survive and some still are in service. Constructed by Romans more than 1,000 years ago, Proserpina Dam in Spain is a notable example. Of a similar age, Kavir Dam, one of the first known arch dams, was built by Mongols in an earthquake prone region of Iran and still survives. The ancients may not have been called engineers, but they acquired the critical faculty of judgment through experience. A respect for shape and mass fueled their ability to advance the profession prior to the arrival of numbers.

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The dawn of using numbers, as we know them, arrived in the 19th century. In 1853 De Sazilly proposed a "profile of equal resistance" for a concrete gravity dam, marking the birth of the "middle third rule."



Figure 3 – Sazilly's Profile



Figure 4 – Furens Dam

Delocre modified De Sazilly's profile to save a bit of masonry and then designed and constructed the 50-meter-high Furens Dam in France in 1858. It still is in service today.ⁱ

Zola Dam was the first arch dam for which the stresses were estimated as if the dam was part of a cylinder. It was completed in France in 1854.

Darcy introduced the theory of flow through porous media in 1856. Embankment dam designs began to evolve with a better understanding of how the designs would have to respond to the prospect of seepage. Lower San Leandro Dam in California was started in 1875 as a zoned embankment with deep cutoffs. It was raised in 1890 to a height of 150 ft and still serves as a reserve water supply. Rockfill dams gained popularity throughout the 20th century. Knowledge about embankment behavior was advanced by Terzaghi (seepage), Casagrande (filters), Fellenius (slope stability), and many others. Many embankment dams were constructed, building upon increasing understanding of how their materials behave, adding to the scope of engineering judgment.

Gravity dam designs continued to improve. Uplift's effect on behavior was recognized and designs began to address the needs for positive cutoff and drainage. In 1914-5, both Elephant Butte and Arrowrock dams were designed with both foundation and internal dam body drains.

In the 1960s several countries introduced ideas for rapid construction of concrete dams. This led to the advent of roller compacted concrete (RCC) construction which the USACE championed

with the construction of Willow Creek Dam in Oregon in the early 1980s. Improvements in design and construction practice since then have promoted RCC as a favored material for concrete dams worldwide.

Arch dam designs gained attention in the late 1800s and then evolved throughout the 20th century. Timoshenko's theory of plates and shells and Boussinesq's theory of elastic half-space fed design improvements. The crown cantilever method of estimating stresses employed a single cantilever with bounding arches, and this method was used to evaluate Bear Valley Dam in California. Variations on the crown cantilever method were applied to many designs, notably Noetzli's designs for USBR's Buffalo Bill and Pathfinder dams. Proud of his achievement, Noetzli applied for and was granted a patent for his design.





Figure 5 – Noetzli's Patent Application

Figure 6 – Mareges Dam

Design innovation continued with advances such as Jorgensen's configuration reducing the radius toward the base, creating a "constant angle" design. Coyne improved on prior designs with Mareges Dam in France and introduced the concept of the double curvature arch dam.

Working at USBR, Howell and Jaiquith formalized the computation for both arches and cantilevers that ultimately became the trial load method.ⁱⁱ

Now we have reliable analytical tools capable of estimating responses to the loads that a dam (or appurtenant works) must safely resist -- provided that we understand the distribution of the properties governing behavior and the mechanics of response. Engineering judgment underpins our understanding.

As improved understanding of how dams respond to loads evolved, so did standards for evaluating dam safety. One of the earliest works to find level ground among all agencies with jurisdiction was the 1979 publication of <u>Federal Guidelines for Dam Safety.ⁱⁱⁱ</u> Subsequently USACE, USBR, FPC (now FERC), and many states produced guidance documents outlining the acceptable margins of safety for responses to loads. Standards-based acceptance criteria were born, and they are still in use as the underlying rationale for determining if a dam is safe enough in its environment. How these criteria are documented varies with the agency having jurisdiction; however, acceptance criteria (required factors of safety) are generally consistent among the agencies. Numbers underpin the analyses employed to compare estimated behavior with acceptance criteria.

In the past few years, agencies with jurisdiction have enhanced traditional standards-based criteria by requiring dam safety decisions to consider risk. To date, USACE and USBR have established risk assessment guidelines. It is important to note that standards-based criteria still lie at the heart of risk-informed decision making. The following excerpt from USACE's ER 11102-1156^{iv} illustrates the role of traditional acceptance criteria:

"The standards-based or essential guidelines approach is included in the risk-informed approach to the dam safety program and dam safety program decisions will now be riskinformed."

UNCERTAINTY

Using numbers demands an appreciation of the uncertainties numbers may present. There are the epistemic uncertainties that involve the limits of knowledge. Uncertainties in the physical conditions of a dam and its foundation are aleatory. Both types of uncertainties are subject to estimates of their correctness and applicability as functions of probability. Probability is purely subjective -- best characterized as one's degree of belief, requiring the exercise of engineering judgment.

Aleatory Uncertainty.

How soil and rock respond to load presents a difficult challenge in evaluating safety. Recent work by Christian and Baecher^v summarized the uncertainties with soils as both aleatory (spatial and temporal variation) and epistemic (parameter error and model bias). Similar uncertainties are applicable to rock. Both soil and rock behave differently in the natural (undisturbed) state and in the fill or excavated (disturbed) states. Depending on the property of interest, either soil or rock may exhibit continuous or discrete variable behavior. Evaluating the safety of an existing dam requires a look-back at the original design properties, the dam's service life during which the original design properties may have changed, and perhaps the need for sampling and testing to inform a decision regarding likely response to a load not previously experienced.

Often the most vexing problem in evaluating safety of an existing dam is the limited amount of data for the properties affecting actual behavior. Data about one dam are not transportable to another dam. Having data without knowing the actual mechanics controlling behavior can lead to constructing erroneous models. This is especially tricky if zones of weakness control behavior.

Epistemic Uncertainty.

There are several ways in which the limits of knowledge affect our ability to evaluate dam safety. Our predictions about extraordinary events rely on our knowledge of the past. In that regard, we are prisoners of history. The White Queen scolds Alice for not understanding to eat jam only on Thursday. To which Alice replies: "It's not Thursday. I can't remember things before they happen." To which the White Queen retorts: "It's a poor sort of memory that only works backward."^{vi}



Figure 7 – Alice and the White Queen

Flood and earthquake predictions look backward to see ahead, but they do not fully recognize the randomness of nature. The engineer making judgments about a dam's safety relies upon estimates of flood and earthquake loadings provided by experts specializing in hydrology and seismicity. The analyses involved in deriving the loading estimates involve a multiplicity of variables that affect the outcomes. There often is stunning disagreement among experts about what the loading estimates should be. Nature regularly provides surprises that exceed prior estimates. More often, centuries pass without loads approaching predictions.

Recognizing the limits to what can be known with certainty requires engineering judgment as the key to making good decisions about dam safety.

We imagine that our memory works both ways, but the reality is we can only use the past to guide our predictions of the future.

Predicting the magnitude and duration of extraordinary events, both floods and earthquakes, rest on past history and their probability of occurrence in the future.

<u>Floods.</u> H. C. Riggs's 1961 paper entitled "Frequency of Natural Events"^{vii} was one of the first flood studies to develop the relationship between magnitude, design life, and probability of exceedance by using a cumulative frequency curve.



Figure 8 – Cumulative Frequency

The current state of practice estimates maximum precipitation and flooding that employs a multitude of continuous and discrete variables. The estimates of precipitation and flow from historical records are examined both deterministically (precipitation) and stochastically (flood).



Figure 9 – Annual Exceedance

Flood recurrence intervals are estimated from cumulative frequency. A dam's safety is then measured as its ability to safely pass a selected Inflow Design Flood, the magnitude and duration of which is based on an evaluation of failure consequences. Floods approaching or exceeding estimated maximums occur (Pearl River 1979, Colorado 2014, South Carolina, 2015). Current estimates may not capture changing climate. They may not consider loss of land cover that decreases runoff time, resulting in floods with higher peaks. Salas *et al* address the difficulty of making reasonable estimates in their review <u>Uncertainty in the PMP and PMF</u>:

"... in recent decades, there has been a growing concern (expressed in literature) regarding the uncertainties involved in estimating such extreme events..."^{viii}

Earthquakes. Dams are designed and maintained to resist earthquakes. Earthquake study probably began after the November 1, 1755, Lisbon Earthquake, a disastrous event with an estimated magnitude approaching 9.0. Through the years, many attempts have been made to predict magnitude and frequency of damaging earthquakes, especially in earthquake-prone areas such as California and China.

Current practice is based on earthquake history and the physical attributes of faults capable of generating ground motions. These are blended in an analysis that combines both deterministic and probabilistic estimates of the extent and duration of earthquake-induced strong shaking from each earthquake source. Conditional mean spectra (CMS) are developed for various estimated recurrence intervals depending on hazard to the public.



Akin to the uncertainties of the PMP and PMF are those of earthquakes. Clarence Allen observed:

"... some of scientists are now challenging this simple concept (elastic rebound), arguing from laboratory experiments and theoretical models as well as from some seismological and geological observations, that the friction on a fault at seismogenic depths (that is, the depth at which rupture initiates is so complicated and so unpredictable, that something akin to **chaotic** behavior may be more typical than **systematic** behavior – not only in the time intervals between large earthquakes, but also in their magnitudes, rupture lengths, and locations along a specific fault system. And this, if true, is certainly not good news for those of us involved in hazard assessment, where the time and size of the last large earthquake has often been an important parameter in our prognostications for the near future."

Allen made those remarks 20 years ago, concluding with a challenge to the strongly held belief that tectonic processes do not change with time. From what we know today, he was prescient.

What Numbers Can Do

Numbers provide the means by which we seek to understand uncertainty, the principal question of which is: "how will these materials behave given the expected loading conditions?"

Numbers estimating properties drive the input. Analyses employ input numbers to produce the output, an estimate of behavior. Measurements, tests, and references providing the basic physical properties such as unit weight, shear strength, permeability, and deformability can be input to generate analyses.

Fundamental differences exist among the properties of concrete, steel, soil, and rock. The properties of concrete and steel do not vary spatially; however, the properties of soil and rock

may vary widely over short distances. How those variations are distributed is a key question for analysis.

Analysis requires a model of the dam and its foundation. For concrete dams, the focus is on the properties of the foundation that promote or threaten stability. An important example is rock wedges capable of movement that may deprive a concrete dam of support. Modeling an embankment focuses not only on the properties of the embankment but also on the foundation. Uncertainty in the physical and spatial distributions of the properties may produce model error. That uncertainty is captured in the required factors of safety for reliability analyses.

<u>Reliability Analyses.</u> The standards-based approach to predicting behavior utilizes both data and estimates of the distribution of the data. The probability of the spatial distribution of properties is based on data and visual observations.

Which properties can we estimate with confidence? The easy ones are unit weights, geometry, and reservoir level.

Which loads can we estimate with confidence? The easy ones are the weight of the dam and the reservoir water. The difficult ones are the loads applied by floods and earthquakes.

Which responses to load can we estimate with confidence? The responses of concrete and steel are not difficult. Their properties are well understood as well as their likely response to loads. Difficulties arise when we connect a dam with the ground and estimate the response of soil and rock.

We estimate the number(s) for each property of interest based on test results and experience with similar materials. If there are no clear differences in the spatial distribution of strengths, a single estimated strength often is applied throughout. Simplification introduces the prospect of error. Analysis becomes difficult if spatial differences are evident because the probabilities of those differences must be taken into account.

Foundation surfaces routinely are simplified as flat and sloping planar surfaces, ignoring important foundation assets that contribute to sliding resistance. Tensile capacity along concrete-rock contacts and roughness are routinely ignored to produce a non-linear model relying on friction alone to resist sliding. "Proving" tensile capacity in the foundation of an existing dam is difficult, expensive, and doubtful of an acceptable resolution because of the work required to obtain a statistically valid sample. Alternatively, the exercise of engineering judgment would suggest that properly placed concrete will strongly adhere to a properly prepared rock surface. An examination of the properties of the foundation and concrete, foundation preparation, and the care taken during construction will support judgment.

<u>Risk Analyses.</u> Steven Vick, in <u>Degrees of Belief</u>, <u>Subjective Probability and Engineering</u> <u>Judgment^x</u>, presents the case for risk analysis as the alternative to reliability analysis.

Risk analysis differs from reliability analysis in several respects, the first being that while it may make use of results from one or more geomechanical models, it does not necessarily rely on

these models directly. Risk analysis provides the ability to address multiple failure modes – not just one – whether geomechanical models are available to represent them or not. It goes beyond where most models stop, to the full progression of failure processes and the factors affecting this progression – be they technical, human, or otherwise."

An analysis identifies an initiator, and then follows the effect of that initiator through any progression of events that could lead to a loss of reservoir control -- the failure mode. At each step in the progression, the probability of continued progression is evaluated against its consequences. The risk is then defined as the product of the probability of failure and its consequences.

Developing reliable geomechanical models is difficult.

Leonard Mladinow observes: "We also use our imagination to take shortcuts to fill gaps in patterns of nonvisual data. As with visual input, we draw conclusions and make judgments based on uncertain and incomplete information, and we conclude, when we are done analyzing the patterns, that our 'picture' is clear and accurate. But is it?"^{xi}

Baecher and Christian highlighted some of the difficulties in their "Geotechnical Reliability – Ten Unresolved Problems." Two of their findings about internal erosion illustrate difficulties, one in the exercise of judgment and another in the limit of knowledge.

- "Typical event trees for internal erosion include physical processes that are intuitively reasonable but unobservable, and add a somewhat theological aspect to predictions."
- "The major problem is that we lack a physical model for predicting internal erosion from first principles. The issue is not one of applying probabilistic methods to a problem whose physics is well understood. It is a problem for which the basic physics of failure has not been described adequately."

Rothman and Sudarshan summarized debates about fundamental physics in words that speak to our state of knowledge: "The innocuous interchange of the words 'theory' and 'model' evidently feeds a confusion of models with the real world. The Earth is treated as a billiard ball until that idealization does not work, then more structure is brought in. Can this process result in the World Equation, the Equation of the Universe? No, it results in equations of the latest model."^{xii}

Risk analysis has gained some acceptance in evaluating dam safety. It is best accomplished by experienced practitioners in the process of expert elicitation. Results from risk analyses performed by inexperienced personnel are not likely to produce reliable outcomes suitable for decision-making.

What Numbers Can't Do

A key element in any dam safety monitoring program is visual surveillance. The likelihood that visual surveillance, augmented by instrumented monitoring, would fail to detect a progression in a potential failure mode is unlikely.

"Monitoring of every dam is mandatory, because dams change with age and may develop defects. There is no substitute for systematic intelligent surveillance. But monitoring and surveillance are not synonymous with instrumentation." Ralph Peck

Experience clearly demonstrates that human error initiates most, if not all, dam incidents and failures.^{xiii} Numbers are not reliable predictors of human behavior. Barring an earthquake or flood, experience demonstrates that a dam safety incident or loss of reservoir control seldom, if ever, occurs without some physical manifestation of the potential failure progression.

A good example is the Big Bay failure in Georgia.^{xiv} The authors noted: "From the time of first filling of the dam, completed in 1993, until the failure in 2004, the dam exhibited several distress indicators and other warning signs." Evaluation of the Big Bay failure cited 11 separate human factors contributory to the failure concluding: "If these combined deficiencies in human factors - which interacted with physical factors in complex ways . . . had not been present, it is very unlikely that the dam would have failed."

Careful review of instrumented measurements is another key to evaluation. Another good example is the recent incident at Wanapum Dam in Washington, where a trend of increasing movement in a monolith presaged the crack that led to the discovery of errors in design and workmanship. Unexpected behavior went unnoticed for several years. Design errors went unnoticed for more than 50 years.

Three contemporary and widely publicized failures are poster children for human error - Swift

No. 2, Taum Sauk, and Silver Lake. All three were regulated projects with long histories of inspections by owners, regulators, and consultants. All three project exhibited evidence of problems long before they failed.

Pat Regan once likened the game of dam safety to a three-legged stool and noted that all three legs (regulator, owner, and consultant) have to function properly to guard safety. Given contemporary experience, one might ask: "Who's minding the store?" Although each project had accepted reliability analyses. none of the failures would have been prevented by risk analysis. Understanding human error and avoiding it is an elusive goal not



Figure 11 – Wanapum Deck Movements

reached by numbers alone. Engineering judgment is required.

James Reason points to the success of the airline industry in creating an organizational culture of safety.^{xv} Creating a culture of safety for every dam requires leadership from every owner. Until more attention and appropriate funding are applied to the issue of reducing human error in dam safety, incidents and failures are likely to continue.

Analytical Error.

Analyses support decision making, and understanding the capabilities and limitations of analyses is essential in making appropriate decisions. There is a tendency among dam safety professionals to ascribe ultimate value to analyses, not recognizing the uncertainties that accompany the analyses.

Hynes and Vanmarcke performed field testing of the reliability of embankment performance prediction that demonstrated the wide spread of expert opinion using the same parameters for analysis.^{xvi} Experience has shown that the likelihood of any analysis being incorrect exceeds the likelihood that it is correct. A recent example demonstrates the lack of agreement produced by differences in analytical approaches.

Faced with prediction of higher ground motions during the design earthquake, an owner commissioned a surge tank stability analysis. The owner's engineer analyzed the expected behavior of the surge tank during an earthquake, concluding that it would fail in base shear. The analysis was submitted to the regulator who rejected it, citing that the mode of failure was moment instability. The owner sought a third opinion. Another consultant was hired and returned a verdict that the surge tank would fail in buckling. One thing became clear: the surge tank is likely to fail in the design earthquake, never mind the mode of failure. Each analyst brings a degree of belief, a personal exercise of engineering judgment based on an evaluation of the facts in evidence. Overconfidence is a weakness not easily overcome.^{xvii}

Building consensus through expert elicitation is favored to reduce the likelihood of analytical error and offer an opportunity to find the best estimate of how a dam will behave under load. The limits of knowledge will always constrain outcomes. Heuristics and biases unrelentingly affect analyses. Analytical results must be viewed with caution. Most dam incidents and failures occur independent of the accuracy of numerical analyses.

Ralph Peck offered an opinion regarding analyses that is worthy of consideration. *Our* concentration on investigating the properties of the materials of which dams are made, and on the technical analysis of the anticipated behavior, should be matched by attention to the nontechnical and human factors that are no less a part of this branch of engineering."

Risk and the Public.

Recent advances in utilizing risk as a guide to decision-making measure the importance of a decision based on the probability of occurrence and the number of fatalities anticipated – the two essential numbers in the fN diagram. Tolerable risk is measured by its position on the fN diagram.



Figure 12 -- Typical fN Diagram^{xviii}

There are two troubling aspects of this approach to dam safety. The first is how risk is conveyed to the folks who are likely to die if a dam fails. A good summary is presented by Herkert in "Ethical Risk Assessment: Valuing Public Perceptions."^{xix}

"The conventional view of risk communication, held by many engineers, is that risk communication need consist merely of "educating" the public to endorse expert judgment concerning which risks are acceptable and which are not. Under this model of risk communication, the experts have a corner on the truth of the matter; the only problem is to see that the public is properly informed of the experts' views."

Paul Slovic in "Perception of Risk" wrote:

"... there is wisdom as well as error in public attitudes and perceptions. Lay people sometimes lack certain information about hazards. However, their basic conceptualization of risk is much richer than that of the experts and reflects legitimate concerns that are typically omitted from

expert risk assessments. As a result, risk communication and risk management efforts are destined to fail unless they are structured as a two-way process."^{xx}

In general, we have an uninformed public, and we have not conveyed risk to the point where the public is prepared to act in the event of an emergency. Many people who are PAR (people at risk) in the analyses don't know that they live downstream from a dam capable of causing immense damage if it fails. Have the PAR participated in establishing the limits of "tolerable risk?"

Lewisville Lake Dam presents a contemporary example of public reaction to the risks to 431,000 PAR from dam failure.

"Dallas doesn't sit along a coast line, but it does face a catastrophic flooding threat from aging Lewisville Dam, about 30 miles north of the city. When Lake Lewisville is full, the dam holds back 2.5 billion tons of water. If the dam were to fail, the loss of life and consequences for North Texas' economy would be a catastrophe of biblical proportions. Walls of water would tear through parts of Lewisville, Carrollton, Farmers Branch, Irving all the way to downtown Dallas, which could be under 50 feet of water.

It's time to square with people. The region deserves a more robust discussion of the dam's condition. Repairing this regional lifeline could cost hundreds of millions of dollars, and North Texas needs to be ready to respond financially. Residents need the chance to understand what's at stake."^{xxi}

Videos of the streets in Tokyo during the aftershocks of the Tohoku earthquake showed pedestrians and traffic calmly observing traffic controls while the buildings all around them were swaying. Would our population respond as calmly or would it panic? The steps to reduce the risk to the public during a serious dam safety emergency beg resolution.

As difficult as it might be, we have an obligation to inform the public we serve about the risk posed by the works we defend. For the most part, we haven't done that.

CONCLUSION

The key to knowing if there is safety in numbers relies on having a clear understanding of what numbers can and cannot do.

Numbers are necessary, but not sufficient, in our duty to hold public safety paramount. Numbers alone, without the benefit of engineering judgment, will not serve that duty.

"While all these new things enable us to do our work smarter, faster, better, and with improved understanding of the materials with which we work, there is one vital, but not so new component of successful geotechnical engineering that is now more important than ever before: that, of course, is sound **engineering judgment**. Despite all the new tools, data and information sources, computational aids, and guidance documents, soils and rock still come in many forms. Their properties may change with time and environment, the boundaries are usually uncertain, no two projects are ever the same, and surprises often lie beneath the surface."^{xxii}

Is there safety in numbers alone? No.

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